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A Remark on Weighted (L^p,L^r)-Boundedness for Rough Multilinear Oscillatory Singular Integrals

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Abstract. This paper studies the weighted (L^p,L^r)-boundedness for a class of multilinear oscillatory singular operators with real-valued polynomial phases and rough homogeneous kernels belonging to $L\log^+ L(S^{n-1})$, and establishes two criteria on the corresponding weighted bounds for such operators.

AMS subject classifications: 42B20, 42B35

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1 Introduction

Let us consider the following multilinear oscillatory singular integral operator T^{A_1,A_2} defined by

$$T^{A_1,A_2}f(x)=\text{p.v.}\int_{\mathbb{R}^n} e^{iP(x,y)}\frac{\Omega(x-y)}{|x-y|^{n+M-1}}\prod_{j=1}^{2}R_{m_j}(A_j;x,y)f(y)dy,$$

and the corresponding truncated operator without phase S^{A_1,A_2} defined by

$$S^{A_1,A_2}f(x)=\text{p.v.}\int_{|x-y|<1}\frac{\Omega(x-y)}{|x-y|^{n+M-1}}\prod_{j=1}^{2}R_{m_j}(A_j;x,y)f(y)dy,$$

where $P(x,y)$ is a real-valued polynomial on $\mathbb{R}^n\times\mathbb{R}^n$, $m_1,m_2\in\mathbb{N}$, $M=m_1+m_2$, $R_{m_j}(A_j;x,y)$ denotes the m_j-th remainder of the Taylor series of A_j at x about y, more precisely,

$$R_{m_j}(A_j;x,y)=A_j(x)-\sum_{|\gamma|<m_j}\frac{1}{\gamma!}D^{\gamma}A_j(y)(x-y)^{\gamma},\quad j=1,2.$$

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Operator of this type has been studied in several works. In 1998, Chen, Hu and Lu [1] studied the case for $\Omega\in L^q(S^{n-1})$ $(1<q\le\infty)$ and A_1 has derivatives of order m_1-1 in $\mathrm{BMO}(\mathbb{R}^n)$, A_2 has derivatives of order m_2 in $L^{r_0}(\mathbb{R}^n)(1<r_0\le\infty)$, and showed that if $P(x,y)$ is a non-degenerate real valued polynomials, then T^{A_1,A_2} is bounded from $L^p(\mathbb{R}^n)$ to $L^r(\mathbb{R}^n)$ if and only if S^{A_1,A_2} is bounded from $L^p(\mathbb{R}^n)$ to $L^r(\mathbb{R}^n)$, where $1/r=1/p+1/r_0$. Subsequently, [8] and [9] studied the special cases of [1] and given the weighted results, respectively. We [19] recently obtained the weighted version of [1]. In addition, Lu and Yan [15] extended the results of [1] to the case $\Omega\in L\log^+L(S^{n-1})$, by assuming that A_1 is a radial function and has derivatives of order m_1-1 in $\mathrm{BLO}(\mathbb{R}^n)$. In this paper, we will extend the results in [15] to weighted cases.

Before formulating our main results, we first recall some relevant definitions.

Definition 1.1 ([7]). (i) *Suppose that $\omega(r)\ge 0$ and $\omega\in L_{\mathrm{loc}}(\mathbb{R}_+)$. For $1<p<\infty$, we say $\omega\in A_p(\mathbb{R}_+)$, if there is a $C>0$ such that for any $I\subset\mathbb{R}_+$,*

$$\left(\frac{1}{|I|}\int_I\omega(r)dr\right)\left(\frac{1}{|I|}\int_I\omega(r)^{-1/(p-1)}dr\right)^{p-1}\le C<\infty.$$

Moreover, if there is a $C>0$ such that

$$\omega^*(r)\le C\omega(r)\quad \text{a.e.}\quad r\in\mathbb{R}_+.$$

then we say $\omega\in A_1(\mathbb{R}_+)$, where ω^* denotes the Hardy-Littlewood maximal function of ω defined by

$$\omega^*(t)=\sup_{t\in I\subset\mathbb{R}_+}\frac{1}{|I|}\int_I\omega(r)dr.$$

(ii) *For $1<p<\infty$, we denote*

$$\widetilde{A}_p(\mathbb{R}_+)=\{\omega:\omega\ge 0,\omega\in L_{\mathrm{loc}}(\mathbb{R}_+)\text{ and }\omega^2\in A_p(\mathbb{R}_+)\}.$$

Definition 1.2 ([15]). *Let $b(r)\in L_{\mathrm{loc}}(\mathbb{R}_+)$. We say $b(r)\in\mathrm{BMO}(\mathbb{R}_+)$, if*

$$\|b\|_{\mathrm{BMO},+}=\sup_{I\subset\mathbb{R}_+}\frac{1}{|I|}\int_I|b(r)-b_I|dr<\infty,$$

where $b_I=|I|^{-1}\int_I b(r)dr$ and I is an interval in $\mathbb{R}_+$.

Definition 1.3 ([5]). *A locally integrable function $a(x)$ will be said to belong to $\mathrm{BLO}(\mathbb{R}^n)$ if there is a constant C such that for any cube Q*

$$m_Q(a)-\inf_{x\in Q}a(x)\le C,$$

where $m_Q(a)=|Q|^{-1}\int_Q a(x)dx$. If $a\in\mathrm{BLO}(\mathbb{R}^n)$, then we denote

$$\|a\|_{\mathrm{BLO}}=\sup_Q\{m_Q(a)-\inf_{x\in Q}a(x)\}.$$