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Efficient Spectral Methods for Transmission Eigenvalues and Estimation of the Index of Refraction

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Abstract. An important step in estimating the index of refraction of electromagnetic scattering problems is to compute the associated transmission eigenvalue problem. We develop in this paper efficient and accurate spectral methods for computing the transmission eigenvalues associated to the electromagnetic scattering problems. We present ample numerical results to show that our methods are very effective for computing transmission eigenvalues (particularly for computing the smallest eigenvalue), and together with the linear sampling method, provide an efficient way to estimate the index of refraction of a non-absorbing inhomogeneous medium.

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1 Introduction

The inverse electromagnetic scattering problem plays an important role in many applications, and is notoriously difficult. Recently a new method using transmission eigenvalues to estimate the index of refraction of a non-absorbing inhomogeneous medium is proposed in [4, 5, 9]. The method consists of several steps. First, the support of the scattering obstacle can be recovered by using the measured scattering data and the linear sampling method [12], and the transmission eigenvalues can be identified from the far field data. Then, the bounds for smallest and largest eigenvalues of the (matrix) index of refraction can be obtained in terms of the support of the scattering obstacle and the first

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transmission eigenvalue of the anisotropic media [3]. Finally, reconstructions of the electric permittivity (if it is a scalar constant) or an estimate of the eigenvalues of the matrix in the case of anisotropic permittivity can be obtained [5].

The effectiveness of the above method rests on having an efficient and robust algorithm for computing transmission eigenvalues for a scalar permittivity. In this paper, we develop efficient spectral methods for computing the transmission eigenvalues in circular and rectangular domains. In particular, for circular domains with stratified media, our method reduces the problem to a sequence of one-dimensional transmission eigenvalue problems that can be solved efficiently and accurately by a spectral-element methods. An error estimate for convergence of the transmission eigenvalues is also provided in this case.

We present ample numerical results to show that our methods are very effective, particularly for computing the few smallest eigenvalues. By using this together with the linear sampling method, we can effectively estimate the (matrix) index of refraction of a non-absorbing inhomogeneous medium.

The organization of the paper is as follows: In §2, we describe the general approach introduced in [4, 5, 9] for estimation of the index of refraction of a non-absorbing inhomogeneous medium. In §3, we derive a weak formulation, construct a Fourier-spectral-element method and derive error estimates for transmission eigenvalues in circular domains. In §4, we describe a spectral method for computing transmission eigenvalues in rectangular domains. We present some numerical results to validate our numerical algorithms in §5.

2 Description of the general approach

In this section, we describe briefly the method introduced in [4, 5, 9] for estimation of the index of refraction of a non-absorbing inhomogeneous medium. We first show how to obtain transmission eigenvalues from far field data, and then describe an algorithm to reconstruct/estimate the index of refraction.

2.1 Transmission eigenvalue problem

Let $D\subset R^d$ $(d=2,3)$ be a bounded, simply connected open set with a piecewise smooth boundary ∂D. We assume that the domain D is the support of an anisotropic dielectric object, and the incident field is a time-harmonic electromagnetic plane wave with frequency ω. Then, the scattering by the anisotropic medium leads to the following problem for the interior electric and magnetic fields E^{int}, H^{int}, and the scattered electric and magnetic field