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Adaptive Runge-Kutta Discontinuous Galerkin Methods with Modified Ghost Fluid Method for Simulating the Compressible Two-Medium Flow

Jun Zhu[1] and Jianxian Qiu[2,*]

[1] *College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 210016, P.R. China.*

[2] *School of Mathematical Sciences and Fujian Provincial Key Laboratory of Mathematical Modeling and High-Performance Scientific Computation, Xiamen University, Xiamen, Fujian 361005, P.R. China.*

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Abstract. In this paper, we investigate using the adaptive Runge-Kutta discontinuous Galerkin (RKDG) methods with the modified ghost fluid method (MGFM) in conjunction with the adaptive RKDG methods for solving the level set function to simulate the compressible two-medium flow in one and two dimensions. A shock detection technique (KXRCF method) is adopted as an indicator to identify the troubled cell, which serves for further numerical limiting procedure which uses a modified TVB limiter to reconstruct different degrees of freedom and an adaptive mesh refinement procedure. If the computational mesh should be refined or coarsened, and the detail of the implementation algorithm is presented on how to modulate the hanging nodes and redefine the numerical solutions of the two-medium flow and the level set function on such adaptive mesh. Extensive numerical tests are provided to illustrate the proposed adaptive methods may possess the capability of enhancing the resolutions nearby the discontinuities inside of the single medium flow region and material interfacial vicinities of the two-medium flow region.

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1 Introduction

In general, algorithms proposed for solving the two-medium compressible flow consist of two parts: One is to treat the material interface accurately and the other is the method

*Corresponding author. *Email addresses:* `zhujun@nuaa.edu.cn` (J. Zhu), `jxqiu@xmu.edu.cn` (J. Qiu)

for simulating the single medium fluid precisely. Kapila *et al.* [13] proposed an important class of five-equation models to solve such two-medium problem. And then, Kreeft and Koren [15] provided a new formulation based on Kapila's model [13] for inviscid, non-heat-conducting and compressible two-fluid flow. Some other references of five-equation models for solving two-medium flow were found in [2,21,22,29]. A relatively crucial difficulty for simulating the compressible two-medium flow is the treatment of the moving material interfaces and their immediate vicinities. In 1999, Fedkiw *et al.* [10] provided a ghost fluid method (GFM) to treat the two-medium flow simulations. Essentially, the GFM makes the interface "invisible" during calculation by defining the ghost cell and the ghost fluid. There are subsequent variants of the original GFM with other applications [1,14]. But it is precisely the manner of treatment of the single medium across the interface in the GFM that may cause numerical inaccuracy when there is a strong shock wave interaction with the interface. Liu *et al.* presented a modified GFM (MGFM) [18–20] to overcome this drawback of the original GFM.

On the other hand, we would like to mention the developing history of the famous RKDG method which is used for solving the single medium fluid. In 1973, Reed and Hill introduced the first discontinuous Galerkin (DG) method [27] in the framework of neutron transport. A major development of the DG method was carried out by Cockburn and Shu in a series of papers [4–8]. They employed the explicit, total variation diminishing or strong stability preserving high order Runge-Kutta time discretizations [32] and DG discretization in space with exact or approximate Riemann solvers as interface fluxes and total variation bounded (TVB) limiter [30] to achieve non-oscillatory properties for strong shocks. In [23,24,37], Qiu *et al.* investigated using DG methods with GFM and MGFM for two-medium flow simulations.

As we known, the solutions of the two-medium flow might have numerous local fluid structures including shock waves, contact discontinuities and rarefaction waves. So we would like to use mesh refining and coarsening procedures to assemble cells in the regions covering such fluid structures in an adaptive manner. The RKDG methods are the finite element methods and easy to deal with an adaptive strategy since the mesh refining and coarsening procedures can be applied without taking into account the continuity restrictions through cell's interface (in single medium) or material interface (in two-medium). Flaherty and his cooperators proposed a series of excellent works [3,9,28] on studying parallel and adaptive finite element methods for simulating conservation laws in single medium flow. Also, recently, Zhu and Qiu gave the procedures of the adaptive RKDG methods for solving hyperbolic conservation laws in [35,36].

In this paper, following the study of [24], we would like to investigate using the adaptive RKDG methods with the MGFM for solving the two-medium flow in one and two dimensions. A shock detection technique called KXRCF method [16] is applied as a troubled cell indicator to identify the cells where the mesh should be refined or coarsened, and the detail of implementation procedures is presented on how to refine or coarse the adaptive mesh with hanging nodes and redefine numerical solutions of the two-medium flow and the level set function on such mesh. The organization of this paper is as follows: