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Complete Convergence for Weighted Sums of Negatively Superadditive Dependent Random Variables

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Abstract. Let $\{X_n, n \geq 1\}$ be a sequence of negatively superadditive dependent (NSD, in short) random variables and $\{a_{nk}, 1 \leq k \leq n, n \geq 1\}$ be an array of real numbers. Under some suitable conditions, we present some results on complete convergence for weighted sums $\sum_{k=1}^{n} a_{nk} X_k$ of NSD random variables by using the Rosenthal type inequality. The results obtained in the paper generalize some corresponding ones for independent random variables and negatively associated random variables.

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1 Introduction

Firstly, let us recall the definitions of negatively associated random variables and negatively superadditive dependent random variables. The concept of negatively associated random variables was introduced by Alam and Saxena [1] and carefully studied by Joag-Dev and Proschan [2] as follows.

Definition 1.1. A finite collection of random variables $X_1, X_2, \cdots, X_n$ is said to be negatively associated (NA) if for every pair of disjoint subsets A_1, A_2 of $\{1, 2, \cdots, n\}$,

$$Cov\{f(X_i : i \in A_1), g(X_j : j \in A_2)\} \leq 0,$$

whenever f and g are coordinatewise nondecreasing such that this covariance exists. An infinite sequence $\{X_n, n \geq 1\}$ is NA if every finite subcollection is NA.

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The next dependence notion is negatively superadditive dependence, which is weaker than negative association. The concept of negatively superadditive dependent random variables was introduced by Hu [3] as follows.

Definition 1.2. (cf. Kemperman [4]) A function $\phi : \mathbb{R}^n \to \mathbb{R}$ is called superadditive if $\phi(\mathbf{x}\vee\mathbf{y})+\phi(\mathbf{x}\wedge\mathbf{y})\geq\phi(\mathbf{x})+\phi(\mathbf{y})$ for all $\mathbf{x},\mathbf{y}\in R^n$, where $\vee$ is for componentwise maximum and $\wedge$ is for componentwise minimum.

Definition 1.3. (cf. Hu [3]) A random vector $\mathbf{X}=(X_1,X_2,\cdots,X_n)$ is said to be negatively superadditive dependent (NSD) if

$$E\phi(X_1,X_2,\cdots,X_n)\leq E\phi(X_1^*,X_2^*,\cdots,X_n^*), \tag{1.1}$$

where $X_1^*,X_2^*,\cdots,X_n^*$ are independent such that X_i^* and X_i have the same distribution for each i and ϕ is a superadditive function such that the expectations in (1.1) exist.

We will introduce the notion of sequences of NSD random variables and arrays of rowwise NSD random variables as follows.

Definition 1.4. A sequence $\{X_n,n\geq 1\}$ of random variables is said to be NSD if for all $n\geq 1$, $(X_1,X_2,\cdots,X_n)$ is NSD.

The concept of NSD random variables was introduced by Hu [3], which was based on the class of superadditive functions. Hu [3] gave an example illustrating that NSD does not imply NA, and Hu posed an open problem whether NA implies NSD. In addition, Hu [3] provided some basic properties and three structural theorems of NSD. Christofides and Vaggelatou [5] solved this open problem and indicated that NA implies NSD. Negatively superadditive dependent structure is an extension of negatively associated structure and sometimes more useful than it and can be used to get many important probability inequalities. Eghbal et al. [6] derived two maximal inequalities and strong law of large numbers of quadratic forms of NSD random variables under the assumption that $\{X_i,i\geq 1\}$ are a sequence of nonnegative NSD random variables with $EX_i^r<\infty$ for all $i\geq 1$ and some $r>1$. Eghbal et al. [7] provided some Kolmogorov inequality for quadratic forms $T_n=\sum_{1\leq i<j\leq n}X_iX_j$ and weighted quadratic forms $Q_n=\sum_{1\leq i<j\leq n}a_{ij}X_iX_j$, where $\{X_i,i\geq 1\}$ is a sequence of nonnegative NSD uniformly bounded random variables. Shen et al. [8] obtained Kolmogorov-type inequality and the almost sure convergence for NSD sequences. Shen et al. [9] established some convergence properties for weighted sums of NSD random variables. Since NSD random variables are much weaker than independent random variables and NA random variables, studying the limit behavior of NSD sequence is of interest. The main purpose of the main is to study the complete convergence for weighted sums of NSD random variables.

The following concept of stochastic domination will be used in the paper.

Definition 1.5. A sequence $\{X_n,n\geq 1\}$ of random variables is said to be stochastically dominated by a random variable X if there exists a positive constant C such that

$$P(|X_n|>x)\leq CP(|X|>x)$$