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Self-adaptive Extrapolated Gauss-Seidel Iterative Methods

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Abstract. In this paper, we consider a self-adaptive extrapolated Gauss-Seidel method for solving the Hermitian positive definite linear systems. Based on optimization models, self-adaptive optimal factor is given. Moreover, we prove the convergence of the self-adaptive extrapolated Gauss-Seidel method without any constraints on optimal factor. Finally, the numerical examples show that the self-adaptive extrapolated Gauss-Seidel method is effective and practical in iteration number.

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Key words: Hermitian positive definite, Gauss-Seidel iteration, self-adaptive, extrapolated, linear systems.

1 Introduction

Consider the large sparse system of linear equations

$$Ax=b, \tag{1.1}$$

where $A=(a_{ij})\in\mathbb{C}^{n\times n}$ is a known nonsingular matrix and $x,b\in\mathbb{C}^n$ are vectors. The splitting iterative method is one of the important way for solving the linear systems (1.1). For any splitting $A=M-N$ with a nonsingular matrix M, the basic splitting iterative method can be expressed as

$$x^{(k+1)}=M^{-1}Nx^{(k)}+M^{-1}b, \quad k=0,1,\cdots. \tag{1.2}$$

Let $T=M^{-1}N$ and $c=M^{-1}b$. Then (1.2) can be also written as

$$x^{(k+1)}=Tx^{(k)}+c, \quad k=0,1,\cdots. \tag{1.3}$$

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Consider the usual splitting of A,

$$A = D - L - U, \tag{1.4}$$

where D is a diagonal matrix, $-L$ and $-U$ are the strictly lower and the strictly upper triangular parts of A, respectively. Taking $M = D - L$ in (1.2), the iteration (1.3) yields the classical Gauss-Seidel iterative method, and the iteration matrix of the Gauss-Seidel method is given by

$$T = (D - L)^{-1} U. \tag{1.5}$$

In order to accelerate the convergence of the Gauss-Seidel iterative method for solving the linear systems (1.1), Davod [4] presented the generalized Gauss-Seidel (GGS) iterative method, i.e., using the splitting $A = T_m - E_m - F_m$, where

$$T_m = (t_{ij}) = \begin{cases} -a_{ij}, & \text{for } |i-j| \le m, \\ 0, & \text{otherwise.} \end{cases}$$

So we can see that the GGS method is essentially block-type Gauss-Seidel method.

Gunawardena et. al [7] first introduced the modified Gauss-Seidel method with preconditioned technique. In particular, when coefficient matrix is an M-matrix, or a Z-matrix, or an H-matrix, many researchers [9,13,15,20,21,23,24] presented also the modifications and improvements of preconditioners to solve the linear systems (1.1). Recently, Kohno et. al [12], Kotakemori et. al [14] and Shen et. al [22] extended Gunawardena, Jain and Snyders' works to more general cases and employed new different modified Gauss-Seidel methods by using the general preconditioner P with various parameters. But these modifications and improvements of Gauss-Seidel iterative method are only theoretical and the numerical examples in those references are only small size problems, and the amount of calculation is rapidly increased with the problem size. And it is the most troubling that it is difficult in choosing many proper parameters for these methods.

With regard to parameters that have a significant effect on the convergence rate of the algorithm, Hadjidimos [8] considered the extrapolation and relaxation methods, Hadjidimos and Yeyios [10] proposed the extrapolated Gauss-Seidel method

$$x^{(k+1)} = [(1-\alpha)I + \alpha T] x^{(k)} + \alpha c, \tag{1.6}$$

where T is given by (1.5). Galanis et. al [6] presented methods that compute optimal relaxation factor in the different case for p-cyclic matrices. Bai et. al [1] and Chen et. al [2] considered the optimal convergence factor and the contraction factors of the GSOR method, respectively. Zhang et. al [27] presents a global relaxed non-stationary multisplitting multi-parameter method, while Migallón et. al [19] proposed non-stationary multisplitting with general weighting matrices.

But the optimal parameters or the weighting matrices of all above-mentioned methods are determined in advance, they are not known to be good or bad, this influences the efficiency of these iterative methods. Fortunately, the papers [26] and [18] have applied