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Modulus-based GSTS Iteration Method for Linear Complementarity Problems

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Abstract. In this paper, a modulus-based generalized skew-Hermitian triangular splitting (MGSTS) iteration method is present for solving a class of linear complementarity problems with the system matrix either being an H_+-matrix with non-positive off-diagonal entries or a symmetric positive definite matrix. The convergence of the MGSTS iteration method is studied in detail. By choosing different parameters, a series of existing and new iterative methods are derived, including the modulus-based Jacobi (MJ) and the modulus-based Gauss-Seidel (MGS) iteration methods and so on. Experimental results are given to show the effectiveness and feasibility of the new method when it is employed for solving this class of linear complementarity problems.

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Key words: GSTS method, modulus-based iteration method, linear complementarity problem, H_+-matrix, symmetric positive definite matrix.

1 Introduction

Consider the following linear complementarity problem

$$\vec{w}:=A\vec{z}+\vec{q}\geq 0, \quad \vec{z}\geq 0 \quad \text{and} \quad \vec{z}^T\vec{w}=0, \tag{1.1}$$

where $A\in\mathbb{R}^{n\times n}$ is a large sparse matrix, $\vec{z}\in\mathbb{R}^n$ is an unknown vector and $\vec{q}=(q_1,q_2,\cdots,q_n)^T \in\mathbb{R}^n$ is a given vector. In the sequel, we abbreviate the linear complementarity problem

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(1.1) as LCP($\vec{q}$,A). The notation "$\geq$" means the componentwise defined partial ordering between two vectors and "T" in the superscript denotes the transpose of a vector.

The LCP($\vec{q}$,A) arises in many scientific computing and engineering applications, e.g., the contact problem, the Nash equilibrium point of a bimatrix game, the free boundary problem and the optimal stopping in Markov chain and so on. For more details, see [6, 17, 19] and the references therein.

To get the numerical solution for the large and sparse LCP($\vec{q}$,A), many efficient methods have been presented based on linear algebraic equations, for example, the projected iterative methods with the system matrix being symmetric positive definite(SPD), symmetric positive semi-definite and diagonally dominant ([1, 16, 18]), the modulus-based iterative method ([2, 4–13, 15, 17, 20, 21]) and so on. The main drawback of the projected methods is that we have to project the iterative solution onto the space $\mathbb{R}^n_+=\{x\in\mathbb{R}^n|x\geq 0\}$, which is a costly and complicated work in actual implementations. Especially, it is much more difficult when the system matrix is nonsymmetric or some zero entries appear on the diagonal position.

Recently, Bai in [5] presented a modulus-based matrix splitting iteration method. The method not only covers the known modulus iteration methods and the corresponding modified variants, but also yields a series of modulus-based relaxation methods. For example, the MJ, the MGS, the modulus-based SOR method (MSOR) ([15]), the modified modulus method ([11]) and the non-stationary extrapolated modulus algorithm ([12]). Besides, if the system matrix is an H_+-matrix, the improved modulus-based matrix splitting iteration method turns to the scaled extrapolated modulus algorithms ([13]) and the two-step modulus-based matrix splitting iteration methods ([20]), respectively.

In this paper, based on the generalized skew-Hermitian triangular splitting (GSTS) iteration method ([14]) and the modulus-based matrix splitting iteration methods ([5]), we present a modulus-based GSTS (MGSTS) iteration method for solving large sparse LCP($\vec{q}$,A). By choosing different parameter matrices, we derive a series of existing and new iterative methods, including MJ, MGS, AMJ(the accelerated MJ), AMGS(the accelerated MGS) and AMSOR (the accelerated MSOR) methods. Experimental results are given to show the effectiveness and feasibility of the new method when it is employed for solving the linear complementarity problems with the system matrix either H_+-matrix with non-positive off-diagonal entries or symmetric positive definite.

The paper is organized as follows. In Section 2, some necessary notations and definitions are introduced, some modulus-based matrix splitting iteration methods are reviewed. Then the MGSTS iteration method for solving large sparse LCP($\vec{q}$,A) is established and some special modulus-based methods are given, respectively. In Section 3, when the system matrix is an H_+-matrix with non-positive off-diagonal entries or a symmetric positive definite matrix, the convergence conditions are presented. In Section 4, numerical examples are given to show the performance of the proposed method. Finally in Section 5, we end this paper with some concluding remarks.