J. Math. Study
doi: 10.4208/jms.v48n3.15.01

Vol. **48**, No. 3, pp. 199-221
September 2015

Theory of Dark Energy and Dark Matter

Marco Hernandez[1], Tian Ma[2] and Shouhong Wang[1,*]

[1] *Department of Mathematics, Indiana University, Bloomington, IN 47405, USA.*
[2] *Department of Mathematics, Sichuan University, Chengdu, 610065, P. R. China.*

Received 26 May, 2015; Accepted 7 August, 2015

Dedicated to Professor Louis Nirenberg on the occasion of his 90th birthday with great admiration and respect.

Abstract. In (T. Ma and S. Wang. Gravitational field equations and theory of dark matter and dark energy, Discrete and Continuous Dynamical Systems, Ser. A, 34(2): 335–366, 2014; arXiv:1206.5078v2), a new set of gravitational field equations are derived based only on 1) the Einstein principle of general relativity, and 2) the principle of interaction dynamics, due to the the presence of dark energy and dark matter. With the field equations, we show that gravity can display both attractive and repulsive behavior, and the dark matter and dark energy are just a property of gravity caused by the nonlinear interactions of the gravitational potential $g_{\mu\nu}$ and its dual field. The main objectives of this paper are two-fold. The first is to study the PID-induced cosmological model, and to show explicitly, as addressed in (T. Ma and S. Wang, Astrophysical dynamics and cosmology, Journal of Mathematical Study, 47(4): 305–378, 2014), that 1) dark matter is due to the curvature of space, and 2) dark energy corresponds to the negative pressure generated by the dual gravitational potential in the field equations, and maintains the stability of geometry and large scale structure of the Universe. Second, for the gravitational field outside of a ball of centrally symmetric matter field, there exist precisely two physical parameters dictating the two-dimensional stable manifold of asymptotically flat space-time geometry, such that, as the distance to the center of the ball of the matter field increases, gravity behaves as Newtonian gravity, then additional attraction due to the curvature of space (dark matter effect), and repulsive (dark energy effect). This also clearly demonstrates that both dark matter and dark energy are just a property of gravity.

AMS subject classifications: 35Q75, 37N20, 83C, 83F

*Corresponding author. *Email addresses:* hernmarc@indiana.edu (Hernandez), matian56@sina.com (Ma), showang@indiana.edu (Wang)

Key words: Dark matter, dark energy, principle of interaction dynamics (PID), gravitational field equations, gravitational interaction formula, stable manifold.

---

1 Introduction

Gravity is one of the four fundamental interactions/forces of Nature, and is certainly the first interaction/force that people studied over centuries, dating back to Aristotle, Galileo, Johannes Kepler, Isaac Newton, and Albert Einstein. It was Albert Einstein who first derived the basic law of gravity — the Einstein gravitational field equations — by postulating the principle of equivalence and the principle of general relativity. In mathematical terms, the principle of equivalence says that the space-time is a 4-dimensional Riemannian manifold $\{\mathcal{M},g_{\mu\nu}\}$ with metric tensor $\{g_{\mu\nu}\}$ of $\mathcal{M}$ being the gravitational potential. The principle of general relativity requires that the law of gravity be independent of general coordinate transformations, and dictates the Einstein-Hilbert functional. The Einstein gravitational field equations are then derived using the least action principle, also called the principle of Lagrangian dynamics.

Dark matter and dark energy phenomena are two important phenomena, which requires a more fundamental examination of the law of gravity [7–10]. Recently, we have shown in [2] that the presence of dark matter and dark energy implies that the variation of the Einstein-Hilbert functional must be taken under energy-momentum conservation constraint, and we call such variation the principle of interaction dynamics (PID). With PID, we have derived the new gravitational field equations:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\frac{8\pi G}{c^4}T_{\mu\nu}-\nabla_\mu\nabla_\nu\varphi, \tag{1.1}$$

supplemented by the energy-momentum conservation:

$$\nabla^\mu\left[\frac{8\pi G}{c^4}T_{\mu\nu}+\nabla_\mu\nabla_\nu\varphi\right]=0. \tag{1.2}$$

Here ϕ is a scalar field defined on M, and needs to be solved together with the Riemannian metric $g_{\mu\nu}$, representing the gravitational potential. Also ∇^μ is the gradient operator on M, $R_{\mu\nu}$ and R are the Ricci and scalar curvatures, and $T_{\mu\nu}$ is the energy-momentum of the baryonic matter in the universe.

With the new gravitational field equations, we have shown in [2] that gravity can display both attractive and repulsive effect, caused by the duality between the *attracting* gravitational field $\{g_{\mu\nu}\}$ and the *repulsive* dual vector field $\{\Phi_\mu\}$, together with their nonlinear interactions governed by the field equations. Consequently, dark energy and dark matter phenomena are simply a property of gravity.

The main objective of this article is to further explore the nature of dark matter and dark energy, in connection with