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Weak Convergence Theorems for Mixed Type Asymptotically Nonexpansive Mappings

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Abstract. The purpose of this paper is to prove some weak convergence theorems for mixed type asymptotically nonexpansive mappings with mean errors in uniformly convex Banach spaces. The results presented in this paper extend the corresponding results in the references.

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Key words: Mixed type asymptotically nonexpansive mappings, uniformly convex Banach space, common fixed point, weak convergence.

1 Introduction and preliminaries

Throughout this work, we assume that E is a real Banach space, E^* is the dual space of E and $J:E\to 2^{E^*}$ is the normalized duality mapping defined by

$$J(x)=\{f\in E^*:\langle x,f\rangle=\|x\|\|f\|\|f\|=\|x\|\},\forall x\in E,$$

where $\langle .,.\rangle$ denotes duality pairing between E and E^*. A single-valued normalized duality mapping is denoted by j.

A Banach space E is said to have a *Fréchet differentiable* norm [1] if for all $x\in U=\{x\in E:\|x\|=1\}$, $\lim_{t\to 0}\frac{\|x+ty\|-\|x\|}{t}$ exists and is attained uniformly in $y\in U$.

A Banach space E is said to have the *Kadec−Klee property* [2] if for every sequence $\{x_n\}$ in E, $x_n\to x$ weakly and $\|x_n\|\to\|x\|$, it follows that $x_n\to x$ strongly.

A Banach space E is said to satisfy *Opial's condition* [3] if for any sequence $\{x_n\}$ of E, $x_n\to x$ weakly as $n\to\infty$ implies that

$$\limsup_{n\to\infty}\|x_n-x\|<\limsup_{n\to\infty}\|x_n-y\|,\quad \forall y\in E,y\neq x.$$

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Let K be a nonempty subset of a real normed linear space E. A mapping $T: K \to K$ is said to be asymptotically nonexpansive [4] if there exists a sequence $\{k_n\} \subset [1,\infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$\|T^n x - T^n y\| \le k_n \|x - y\|$$

for all $x,y \in K$ and $n \ge 1$.

A subset K of a real Banach space E is called a retract of E [5] if there exists a continuous mapping $P: E \to K$ such that $Px = x$ for all $x \in K$. Every closed convex subset of a uniformly convex Banach space is a retract. A mapping $P: E \to E$ is called a retraction if $p^2 = p$. It follows that if a mapping P is a retraction, the $Py = y$ for all y in the range of P.

Definition 1.1. [5] Let K be a nonempty subset of a real normed linear space E. Let $P: E \to K$ be a nonexpansive retraction of E onto K. A nonself-mapping $T: K \to E$ is said to be *asymptotically nonexpasive* if there exists a sequence $\{k_n\} \subset [1,\infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \le k_n \|x - y\|$$

for all $x,y \in K$ and $n \ge 1$.

Let K be a nonempty closed convex subset of a real uniformly convex Banach space E. Chidume *et al.* [5] studied the following iteration scheme:

$$\begin{cases} x_1 \in K, \\ x_{n+1} = P((1-a_n)x_n + a_n T(PT)^{n-1} x_n) \end{cases} \tag{1.1}$$

for each $n \ge 1$, where $\{a_n\}$ is a sequence in $(0,1)$ and P is a nonexpansive retraction of E onto K, and prove some strong and weak convergence theorems for asymptotically nonexpansive mapping.

In 2006, Wang [6] generalized the iteration process (1.1) as follows:

$$\begin{cases} x_1 \in K, \\ x_{n+1} = P((1-a_n)x_n + a_n T_1(PT_1)^{n-1} y_n), \\ y_n = P((1-b_n)x_n + b_n T_2(PT_2)^{n-1} x_n) \end{cases} \tag{1.2}$$

for each $n \ge 1$, where $T_1, T_2: K \to E$ are two asymptotically nonexpansive nonself-mappings and $\{a_n\}, \{b_n\}$ are real sequences in [0,1), and prove some strong and weak convergence theorems for asymptotically nonexpansive mappings.

In 2012, Guo *et al.* [7] generalized the iteration process(1.2) as follows:

$$\begin{cases} x_1 \in K, \\ x_{n+1} = P((1-a_n)S_1^n x_n + a_n T_1(PT_1)^{n-1} y_n), \\ y_n = P((1-b_n)S_2^n x_n + b_n T_2(PT_2)^{n-1} x_n) \end{cases} \tag{1.3}$$