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Numerical Approximations of the Spectral Discretization of Flame Front Model

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Abstract. In this paper, we consider the numerical solution of the flame front equation, which is one of the most fundamental equations for modeling combustion theory. A schema combining a finite difference approach in the time direction and a spectral method for the space discretization is proposed. We give a detailed analysis for the proposed schema by providing some stability and error estimates in a particular case. For the general case, although we are unable to provide a rigorous proof for the stability, some numerical experiments are carried out to verify the efficiency of the schema. Our numerical results show that the stable solution manifolds have a simple structure when β is small, while they become more complex as the bifurcation parameter β increases. At last numerical experiments were performed to support the claim the solution of flame front equation preserves the same structure as K-S equation.

AMS subject classifications: 65N35, 65T50, 65L12, 65M70.

Key words: Flame front equation, Finite difference, Fourier method, Error estimates.

1 Introduction

The reduction of a free-interface problem to an explicit equation for the interface dynamics is a challenging issue and much research on this subject has been carried out during decade. A paradigm of two-dimensional problem in combustion theory is the model for the Near Equidiffusive Flames (NEF), which was introduced by Matkowsky

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and Sivashinsky [17] in 1979, this equation is characterized by near-unity Lewis numbers, near-adiabatic flame temperatures and near-uniform enthalpies. For the moving flame front, defined by $x=\xi(t,y)$, the temperature θ and enthalpy S, one ends up with the free-interface problem:

$$\frac{\partial\theta}{\partial t}=\Delta\theta,\ x<\xi(t,y), \tag{1.1}$$

$$\theta=1,\ x\geq\xi(t,y), \tag{1.2}$$

$$\frac{\partial S}{\partial t}=S-\alpha\Delta\theta,\ x\neq\xi(t,y). \tag{1.3}$$

At the front, θ and S are continuous and the following jump conditions occur for the normal derivatives:

$$\left[\frac{\partial\theta}{\partial n}\right]=-\exp(S),\qquad \left[\frac{\partial S}{\partial n}\right]=\alpha\left[\frac{\partial\theta}{\partial n}\right], \tag{1.4}$$

where α is the reduced Lewis number. This equation provides a convenient framework for the theoretical study of a number of flame phenomena. Sivashinsky [21] considered the constant-density model of a premixed flame, and he derived an asymptotic Kuramoto-Sivashinsky (K-S) equation, which describes the evolution of the disturbed flame front:

$$\partial_\tau\Phi+4\partial_\eta^4\Phi+\partial_\eta^2\Phi+\frac{1}{2}(\partial_\eta\Phi)^2=0.$$

Brauner and Lunardi [7] proved instability of the planar travelling wave solution in a two-dimensional free boundary problem stemmed by the propagation of premixed flames. Lorenzi [16] dealt with a free boundary problem, modelling the propagation of premixed flames in a infinite strip in $\mathbb{R}^2$, he proved existence, uniqueness, regularity results for the solutions near the travelling wave. Ducrot and Marion [11] considered a semi-linear elliptic system in combustion theory. In particular, they proved the existence of travelling wave solutions for high activation energy. Brauner *et al.* [5] obtained a simplified quasisteady version of NEF, this simplifiecation allows near the planar front, an explicit derivation of the front equation. And they introduced a parameter ε, rescale both the dependent and independent variables, and proved rigourously the convergence to the solution of the K-S equation as $\varepsilon\to 0$. In [2], Berestycki *et al.* presented a Burgers-Sivashinsky (B-S) equation, a model pertinent to the flame front dynamics subject to the buoyancy effect. Later, Brauner *et al.* [4] derived a quasi-steady (Q-S) equation, a quasi-steady version of the κ-θ in flame theory. Both of two dissipative systems have similar dynamics. A previous work [6] by Brauner *et al.* were concerned with a generalized K-S equation, which is a nonlinear wave equation with a strong damping operator. There are alternative possibilities for reduction of the free-interface problem (1.1)-(1.4) to an explicit equation of the flame front. In other words, starting from the same configuration,