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Spectra of Corona Based on the Total Graph

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Abstract. For two simple connected graphs G_1 and G_2, we introduce a new graph operation called the total corona $G_1 \circledast G_2$ on G_1 and G_2 involving the total graph of G_1. Subsequently, the adjacency (respectively, Laplacian and signless Laplacian) spectra of $G_1 \circledast G_2$ are determined in terms of these of a regular graph G_1 and an arbitrary graph G_2. As applications, we construct infinitely many pairs of adjacency (respectively, Laplacian and signless Laplacian) cospectral graphs. Besides we also compute the number of spanning trees of $G_1 \circledast G_2$.

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1 Introduction

In this paper, all graphs considered are finite, simple connected graphs. Let $G=(V,E)$ be a graph with vertex set $V=\{v_1,v_2,\ldots,v_n\}$ and edge set $E(G)$. The adjacency matrix of G is an $n\times n$ matrix whose (i,j)-entry is 1 if v_i and v_j are adjacent in G and 0 otherwise, denoted by $A(G)$. The degree of v_i in G is denoted by $d_i=d_G(v_i)$. Let $D(G)$ be the diagonal degree matrix of G which diagonal entries are $d_1,d_2,\ldots,d_n$. The Laplacian matrix $L(G)$ of G is defined as $D(G)-A(G)$. The signless Laplacian matrix of G is defined as $Q(G)=D(G)+A(G)$. For an $n\times n$ matrix M associated to G, the characteristic polynomial $\det(xI_n-M)$ of M is called the M-characteristic polynomial of G and is denoted by $\phi(M;x)$. I_n denotes the identity matrix. The roots of $\phi(M;x)$ are called the eigenvalues of matrix M. The set of all eigenvalues is called the spectrum of matrix M or graph G. In particular, if M is the adjacency matrix $A(G)$ of G, then the A-spectrum of G is denoted by

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$\sigma(A(G))=(\lambda_1(G),\lambda_2(G),\ldots,\lambda_n(G))$, where $\lambda_1(G)\geq\lambda_2(G)\geq\cdots\geq\lambda_n(G)$ are the eigenvalues of $A(G)$. If M is the Laplacian matrix $L(G)$ of G, then the L-spectrum of G is denoted by $\sigma(L(G))=(\mu_1(G),\mu_2(G),\ldots,\mu_n(G))$, where $\mu_1(G)\leq\mu_2(G)\leq\cdots\leq\mu_n(G)$ are the eigenvalues of $L(G)$. If M is the signless Laplacian matrix $Q(G)$ of G, then the Q-spectrum of G is denoted by $\sigma(Q(G))=(\nu_1(G),\nu_2(G),\ldots,\nu_n(G))$, where $\nu_1(G)\leq\nu_2(G)\leq\cdots\leq\nu_n(G)$ are the eigenvalues of $Q(G)$. For more review about the A-spectrum, L-spectrum and Q-spectrum of G, readers may refer to [4–7] and the references therein.

It is of interest to study some spectral properties of certain composite operations between two graphs such as the Cartesian product, the Kronecker product, the corona, the edge corona, the neighbourhood corona, the subdivision-vertex neighbourhood corona, the subdivision-edge neighbourhood corona. For example, the A-spectra, L-spectra and Q-spectra of the (edge) corona of two graphs can be expressed by these of the two factor graphs [1–3, 8, 9, 11, 13–17]. Recently, the R-vertex (neighbourhood) corona and R-edge (neighbourhood) corona of two graphs have been defined in [12] and the A-spectra, L-spectra and Q-spectra of these four operations of two graphs were computed in [12].

Motivated by the works above, we define a new graph operation based on the total graph as follows. The total graph [6] of a graph G, denoted by $T(G)$, is that graph whose set of vertices is the union of the set of vertices and the set of edges of G, with two vertices of $T(G)$ being adjacent if and only if the corresponding elements of G are adjacent or incident.

Definition 1.1. The total corona of G_1 and G_2, denoted by $G_1 \circledast G_2$, is obtained by taking one copy of $T(G_1)$ and $|V(G_1)|$ copies of G_2, and joining the ith vertex of G_1 to every vertex in the ith copy of G_2.

Let P_n be a path of order n. Figure 1 depicts the total corona $P_3 \circledast P_2$ of P_3 and P_2. Note that if G_1 is an r-regular graph on n_1 vertices and m_1 edges, and G_2 is an arbitrary graph on n_2 vertices and m_2 edges, then $G_1 \circledast G_2$ has $n_1+m_1+n_1n_2$ vertices and $n_1m_2+n_1n_2+3m_1+\frac{n_1r(r-1)}{2}$ edges.

In this paper, we focus on determining the A-spectra, L-spectra and the Q-spectra of $G_1 \circledast G_2$ in terms of the corresponding spectra of a regular graph G_1 and an arbitrary graph G_2. As applications of these results, we construct infinitely many pairs of adjacency (respectively, Laplacian and signless Laplacian) cospectral graphs. Moreover, we also compute the number of spanning trees of $G_1 \circledast G_2$ in terms of the L-spectra of two factor graphs G_1 and G_2.

2 Main results

In this section, we determine the spectra of total corona with the help of the coronal of a matrix. The M-coronal $\Gamma_M(x)$ of a matrix M of order n is defined [3, 16] to be the sum of