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Spectral Theory of Differential Operators and Energy Levels of Subatomic Particles

Tian Ma[1] and Shouhong Wang[2,*]

[1] *Department of Mathematics, Sichuan University, Chengdu, Sichuan Province, P. R. China.*
[2] *Department of Mathematics, Indiana University, Bloomington, IN 47405, USA.*

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Abstract. Motivated by the Bohr atomic model, in this article we establish a mathematical theory to study energy levels, corresponding to bounds states, for subatomic particles. We show that the energy levels of each subatomic particle are finite and discrete, and corresponds to negative eigenvalues of the related eigenvalue problem. Consequently there are both upper and lower bounds of the energy levels for all subatomic particles. In particular, the energy level theory implies that the frequencies of mediators such as photons and gluons are also discrete and finite. Both the total number N of energy levels and the average energy level gradient (for two adjacent energy levels) are rigorously estimated in terms of certain physical parameters. These estimates show that the energy level gradient is extremely small, consistent with the fact that it is hard to notice the discrete behavior of the frequency of subatomic particles.

AMS subject classifications: 35Q75, 37N20, 83C, 83F

Key words: Spectrum of differential operators, energy levels, Dirac operator, Weyl operator, subatomic particles.

1 Introduction

This article is part of a research program initiated in the last few years by the authors to derived experimentally verifiable laws of Nature based only on a few fundamental first principles, guided by experimental and observation evidences. The new theory we have established gives rise to solutions and explanations to a number of longstanding mysteries in modern theoretical physics. This work is synthesized in a recent by by the authors [8].

Basically, we have discovered three fundamental principles: the principle of interaction dynamics (PID) [5], the principle of representation invariance (PRI) [6], and the principle of symmetry-breaking (PSB) for unification [8].

*Corresponding author. *Email addresses:* `matian56@sina.com` (T. Ma), `showang@indiana.edu` (S.-H. Wang)

PID takes the variation of the action under the energy-momentum conservation constraints, and is required by the dark matter and dark energy phenomena for gravity), by the quark confinements (for strong interaction), and by the Higgs field (for the weak interaction).

PRI requires that the gauge theory be independent of the choices of the representation generators. These representation generators play the same role as coordinates, and in this sense, PRI is a coordinate-free invariance/covariance, reminiscent of the Einstein principle of general relativity. In other words, PRI is purely a logic requirement for the gauge theory.

PSB offers an entirely different route of unification from the Einstein unification route which uses large symmetry group. The three sets of symmetries — the general relativistic invariance, the Lorentz and gauge invariances, as well as the Galileo invariance — are mutually independent and dictate in part the physical laws in different levels of Nature. For a system coupling different levels of physical laws, part of these symmetries must be broken.

This article is motivated 1) by the classical atomic energy level theory, 2) the weakton model of elementary particles [7], and 3) the new field theory for the four fundamental interactions [5,6].

The classical atomic energy level theory demonstrates that there are finite number of energy levels for an atom given by $E_n = E_0 + \lambda_n$, $n = 1,\cdots,N$, where λ_n are the negative eigenvalues of the Schrödinger operator, representing the bound energies of the atom, holding the orbital electrons, due to the electromagnetism.

The weakton model of elementary particles and the unified field theory are developed recently by the authors [5–7]. The field theory is based on two recently postulated principles by the authors: the principle of interaction dynamics (PID) and the principle of representation invariance (PRI). Intuitively, PID takes the variation of the action under energy-momentum conservation constraint. PID offers a completely different and natural way of introducing Higgs fields, and is also required by the quark confinement. For gravity, we show that PID is the direct consequence of Einsteins principle of general relativity and the presence of dark matter and dark energy. PRI requires that the $SU(N)$ gauge theory be independent of representations of $SU(N)$. PRI has remarkably rich physical consequences.

The main objectives of this article are 1) to introduce the energy levels for all subatomic particles, 2) to develop a mathematical theory to study energy levels, and 3) to derive physical implications and predictions of the theory.

Hereafter we explore the key ingredients and the main results in this article.

FIRST, we develop a mathematical theory for estimating the number of negative eigenvalues for a class of differential operators, and consequently the theory is applied to study the energy levels for subatomic particles.

SECOND, the constituents of subatomic particles are spin-$\frac{1}{2}$ fermions, which are bound