J. Math. Study
doi: 10.4208/jms.v50n4.17.06

Vol. **50**, No. 4, pp. 375-390
December 2017

Weak Convergence Theorems for Mixed Type Total Asymptotically Nonexpansive Mappings in Uniformly Convex Banach Spaces

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Received April 24, 2016; Accepted October 19, 2017

Abstract. In this paper, we study a new two-step iteration scheme of mixed type for two total asymptotically nonexpansive self mappings and two total asymptotically nonexpansive non-self mappings and establish some weak convergence theorems in the framework of uniformly convex Banach spaces. Our results extend and generalize several results from the current existing literature.

AMS subject classifications: 47H09, 47H10, 47J25.

Key words: Total asymptotically nonexpansive self and non-self mapping, mixed type iteration scheme, common fixed point, uniformly convex Banach space, weak convergence.

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1 Introduction and preliminaries

Let C be a nonempty subset of a real Banach space E and $T: C \to C$ a nonlinear mapping. $F(T)$ denotes the set of fixed points of the mapping T, that is, $F(T) = \{x \in C : Tx = x\}$, $F = F(S_1) \cap F(S_2) \cap F(T_1) \cap F(T_2)$ denotes the set of common fixed points of the mappings S_1, S_2, T_1 and T_2 and $\mathbb{N}$ denotes the set of all positive integers.

Definition 1.1. A mapping T is said to be total asymptotically nonexpansive [1] if

$$\|T^n(x) - T^n(y)\| \leq \|x - y\| + \mu_n \psi(\|x - y\|) + \nu_n, \tag{1.1}$$

for all $x, y \in C$ and $n \in \mathbb{N}$, where $\{\mu_n\}$ and $\{\nu_n\}$ are nonnegative real sequences such that $\mu_n \to 0$ and $\nu_n \to 0$ as $n \to \infty$ and a strictly increasing continuous function $\psi: [0,\infty) \to [0,\infty)$ with $\psi(0) = 0$.

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From the definition, we see that the class of total asymptotically nonexpansive mappings include the class of asymptotically nonexpansive mappings as a special case; see also [4] for more details.

Remark 1.1. From the above definition, it is clear that each asymptotically nonexpansive mapping is a total asymptotically nonexpansive mapping with $\nu_n = 0$, $\mu_n = k_n - 1$ for all $n \geq 1$, $\psi(t) = t$, $t \geq 0$.

Definition 1.2. A subset C of a Banach space E is said to be a retract of E if there exists a continuous mapping $P\colon E \to C$ (called a retraction) such that $P(x) = x$ for all $x \in C$. If, in addition P is nonexpansive, then P is said to be a nonexpansive retract of E.

If $P\colon E \to C$ is a retraction, then $P^2 = P$. A retract of a Hausdorff space must be a closed subset. Every closed convex subset of a uniformly convex Banach space is a retract.

Definition 1.3. Let C be a nonempty closed convex subset of a Banach space E. A non-self mapping $T\colon C \to E$ is said to be total asymptotically nonexpansive [18] if there exist sequences $\{\mu_n\}$ and $\{\nu_n\}$ in $[0,\infty)$ with $\mu_n \to 0$ and $\nu_n \to 0$ as $n \to \infty$ and a strictly increasing continuous function $\psi\colon [0,\infty) \to [0,\infty)$ with $\psi(0) = 0$ such that

$$\|T(PT)^{n-1}(x) - T(PT)^{n-1}(y)\| \leq \|x-y\| + \mu_n \psi(\|x-y\|) + \nu_n, \tag{1.2}$$

for all $x,y \in C$ and $n \in \mathbb{N}$.

For the sake of convenience, we restate the following concepts and results.

Let E be a Banach space with its dimension greater than or equal to 2. The modulus of convexity of E is the function $\delta_E(\varepsilon)\colon (0,2] \to [0,1]$ defined by

$$\delta_E(\varepsilon) = \inf\Big\{1 - \|\frac{1}{2}(x+y)\| : \|x\| = 1, \|y\| = 1, \varepsilon = \|x-y\|\Big\}.$$

A Banach space E is uniformly convex if and only if $\delta_E(\varepsilon) > 0$ for all $\varepsilon \in (0,2]$.

Definition 1.4. Let $\mathcal{S} = \{x \in E : \|x\| = 1\}$ and let E^* be the dual of E, that is, the space of all continuous linear functionals f on E. The space E has:

(i) Gâteaux differentiable norm if

$$\lim_{t \to 0} \frac{\|x+ty\| - \|x\|}{t}$$

exists for each x and y in $\mathcal{S}$.

(ii) Fréchet differentiable norm [14] if for each x in $\mathcal{S}$, the above limit exists and is attained uniformly for y in $\mathcal{S}$ and in this case, it is also well-known that

$$\langle h, J(x)\rangle + \frac{1}{2}\|x\|^2 \leq \frac{1}{2}\|x+h\|^2 \leq \langle h, J(x)\rangle + \frac{1}{2}\|x\|^2 + b(\|x\|) \qquad (*)$$