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Conditional Residual Lifetimes of $(n-k+1)$-out-of-n Systems with Mixed Erlang Components

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Abstract. We consider an $(n-k+1)$-out-of-n system with component lifetimes being correlated. The main objective of this paper is to study the conditional residual lifetime of an $(n-k+1)$-out-of-n system, given that at a fixed time a certain number of components have failed, assuming that the component lifetimes follow a multivariate Erlang mixture. Comparison studies of the stochastic ordering of the $(n-k+1)$-out-of-n system are presented. Several examples are presented to illustrate and confirm our results.

AMS subject classifications: 62G03, 62P30, 62H20

Key words: Conditional mean residual lifetime, multivariate Erlang mixture, $(n-k+1)$-out-of-n system, dependence structure, exchangeable variables.

1 Introduction

An $(n-k+1)$-out-of-n system is a system such that it consists of n components and works if and only if at least $(n-k+1)$ out of the n components are operating $(k \le n)$. Thus, this system fails if k or more of its components fail. If $k=1$ the system is a series system, and if $k=n$ the system is a parallel system. The system is often considered in the industrial and survival analysis context. Denote the lifetimes of the individual components by $X_1, X_2, \cdots, X_n$ and let $X_{1:n} \le X_{2:n} \le \cdots \le X_{n:n}$ be the corresponding order statistics. Then the lifetime of the $(n-k+1)$-out-of-n system will be represented by the kth order statistic $X_{k:n}$. Let X denote the lifetime of a component of a system. Then $X_t = (X-t|X>t)$ may be interpreted as the residual lifetime of the system at time t, given that the system is alive at time t.

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In recent years, the residual lifetime of an $(n-k+1)$-out-of-n system has been studied extensively. [2] considered the residual lifetime of an $(n-k+1)$-out-of-n system under the condition that at most $(l-1)$ components have failed at time t, i.e., $(X_{k:n}-t|X_{l:n}>t), 1\leq l\leq k\leq n$. [14] extended the concept to an $(n-k+1)$-out-of-n system under the condition that at least j components have failed but the lth failure has not occurred yet at time t, i.e., $(X_{k:n}-t|X_{j:n}\leq t<X_{l:n}), 1\leq j<l\leq k\leq n$. Similar research can be found in [1], [3], [13], and [22].

The research in this area in general focuses on the distribution function, the mean residual lifetime (MRL) and stochastic ordering properties of residual lifetimes under the assumption that the components of a system are independent. See [8], [9], [18], and references therein. In many real situations however, there may be a structural dependence among components of the system. As a result, there are several recent studies considering the dependence among the components. For example, [15] adopted Archimedean copula to reflect the dependence among the components. Others may be found in [7], [17], [18] and references therein.

In this paper, we study the conditional mean residual lifetime function and stochastic ordering properties of an $(n-k+1)$-out-of-n system with assumption that the lifetimes have a multivariate Erlang mixture. The multivariate Erlang mixture is a useful model as it can capture the dependence structure of a large number of multiple variables well. Compared with copula method that is a dominant choice to model multivariate data these days, a multivariate Erlang mixture is more flexible in terms of dependence structure and has a wide range of dependence. Furthermore, it is easy to deal with high dimensional data with a multivariate Erlang mixture, while a copula approach may become much more difficult to use for higher dimensional data. See [12], [19], [20] and references therein. Hence the results in this paper may be useful when the components of a system are of strong dependency and the number of components is high.

Each marginal of a multivariate Erlang mixtures can be viewed as a compound exponential distribution. In this paper, we show that if the counting random variables satisfy the multivariate totally positive of order 2 (MTP_2) property, then the conditional residual lifetime $X^t_{k,j,l,n}=(X_{k:n}-t|X_{j:n}\leq t<X_{l:n}), 1\leq j<l\leq k\leq n$ is stochastically non-decreasing with respect to j and k and non-increasing with respect to l. These properties are consistent with the results when the component lifetimes are independent.

This paper is organized as follows. In Section 2, we present some properties of exchangeable variables with the joint distribution being a multivariate Erlang mixture. The purpose of the section is to simplify the proofs in following sections. In Section 3, we study the conditional mean residual lifetime of an $(n-k+1)$-out-of-n system under the assumption that the lifetimes of the components follow an Erlang mixture. In Section 4, we stochastically compare the conditional residual lifetimes of the $(n-k+1)$-out-of-n system with respect to its various parameters. We conclude Section 5 with some remarks.