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Hybrid WENO Schemes with Lax-Wendroff Type Time Discretization

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Abstract. In this paper, we investigate the performance of a class of the hybrid weighted essentially non-oscillatory (WENO) schemes with Lax-Wendroff time discretization procedure using different indicators for hyperbolic conservation laws. The main idea of the scheme is to use some efficient and reliable indicators to identify discontinuity of solution, then reconstruct numerical flux by WENO approximation in discontinuous regions and up-wind linear approximation in smooth regions, hence reducing computational cost but still maintaining non-oscillatory properties for problems with strong shocks. Numerical results show that the efficiency and robustness of the hybrid WENO-LW schemes.

AMS subject classifications: 65M60, 35L65

Key words: Time discretization methods, WENO approximation, troubled-cell indicator, Hyperbolic conservation laws, Hybrid schemes.

1 Introduction

In this paper, we investigate the performance of a class of hybrid weighted essentially non-oscillatory (WENO) schemes with Lax-Wendroff time discretization, termed as hybrid WENO-LW schemes, with different discontinuity indicators for solving hyperbolic conservation laws:

$$\begin{cases} u_t+\nabla\cdot f(u)=0, \\ u(x,0)=u_0(x). \end{cases} \tag{1.1}$$

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The first finite volume WENO scheme was constructed by Liu *et al.* [16], and the third and fifth-order finite difference WENO schemes in multi-space dimensions were presented by Jiang and Shu [12], in which they setup a framework to compute the smoothness indicators and nonlinear weights which is the key of WENO schemes in the combination of lower order flux to obtain a higher order approximation. Further Balsara and Shu [1] and Gerolymos *et al.* [8] extended the WENO schemes to higher order. The weights for combination of lower order flux is important for WENO approximation. For the case of system, WENO schemes use local characteristic decompositions and flux splitting to avoid or reduce spurious oscillation. But the calculations of nonlinear weights and local characteristic decomposition are expensive. To overcome these drawbacks, Jiang and Shu [12] computed the nonlinear weights from pressure or entropy instead of the characteristic values for Euler equations. Pirozzoli [17] developed an efficient hybrid compact-WENO scheme, which used compact up-wind schemes to treat smooth regions of the flow field and WENO schemes to handle discontinuities regions. Hill and Pullin [11] developed a hybrid scheme which combines the tuned center-difference schemes with WENO schemes, hence achieving automatically the nonlinear weights for WENO schemes in regions of smooth flow away from shocks. But a switch was still necessary for the schemes. Li and Qiu [15] developed hybrid WENO schemes with Runge-Kutta time discretization which combine pure WENO schemes with simple upwind linear schemes, in which they investigated using the different troubled-cell indicators which are borrowed from discontinuous Galerkin (DG) schemes as switches to identify where WENO approximation or upwind linear approximation is applied.

The main idea of hybrid WENO schemes is using WENO approximation in discontinuity and other efficient approximation such as up-wind linear one in smooth region of solution to reduce computational cost. The troubled-cell indicators which can identify where is discontinuity of the solution are key components of hybrid WENO schemes. In [15], Li and Qiu had investigated using the different troubled-cell indicators to identify discontinuity of the solution. There are many troubled-cell indicators based on limiters of DG schemes which are listed by Qiu and Shu [20]. Among them, the total variation bounded (TVB) limiter [4–7] borrowed from the finite volume methodology is a slope limiter based on minmod function. Biswas, Devine and Flaherty (BDF) investigated the moment-based limiter [2] and Burbeau, Sagaut and Bruneau (BSB) investigated an improved moment limiter [3]. Krivodonova *et al.* (KXRCF) designed a limiter [13] to detect discontinuities for DG methods based on super convergent property at the outflow boundary in smooth regions. There are also many other troubled-cell indicators borrowed from finite volume and finite difference methodology, such as the monotonicity-preserving (MP) limiter [25], and modifications of MP (MMP) limiter [22]. Qiu and Shu [21] used some limiters as troubled-cell indicators for Runge-Kutta discontinuous Galerkin (RKDG) methods with WENO limiters to compare their performance. And Zhu and Qiu [28] used these troubled-cell indicators for adaptive RKDG methods. Li and Qiu [15] applied them in hybrid WENO with Runge-Kutta (RK) time discretization schemes to identify where WENO approximation or upwind linear approximation