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A Dimensional Splitting Method for 3D Elastic Shell with Mixed Tensor Analysis on a 2D Manifold Embedded into a Higher Dimensional Riemannian Space

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Abstract. In this paper, a mixed tensor analysis for a two-dimensional (2D) manifold embedded into a three-dimensional (3D) Riemannian space is conducted and its applications to construct a dimensional splitting method for linear and nonlinear 3D elastic shells are provided. We establish a semi-geodesic coordinate system based on this 2D manifold, providing the relations between metrics tensors, Christoffel symbols, covariant derivatives and differential operators on the 2D manifold and 3D space, and establish the Gateaux derivatives of metric tensor, curvature tensor and normal vector and so on, with respect to the surface $\vec{\Theta}$ along any direction $\vec{\eta}$ when the deformation of the surface occurs. Under the assumption that the solution of 3D elastic equations can be expressed in a Taylor expansion with respect to transverse variable, the boundary value problems satisfied by the coefficients of the Taylor expansion are given.

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1 Introduction

A shell is a three-dimensional (3D) elastic body that is geometrically characterized by its middle surface and its small thickness. The middle surface $\Im$ is a compact surface in $\Re^3$ that is not a plane (otherwise the shell is a plate), and it may or may not have a boundary. For instance, the middle surface of a sail has a boundary, whereas that of a basketball has no boundary.

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At each point $s \in \Im$, let $n(s)$ denote a unit vector normal to $\Im$. Then the reference configuration of the shell, (that is, the subset of $\Re^3$ that occupies before forces are applied to it), is a set of the form $\{(\vec{\theta}+\xi n(s)) \in \Re^3 : \vec{\theta} \in \Im, |\xi| \leq e(\vec{\theta})\}$, where the function e: $\Im \to \Re$ is sufficiently smooth and satisfies $0 \leq e(\vec{\theta}) \leq \varepsilon$ for all $s \in \Im$. Additionally, $\varepsilon > 0$ is thought of as being 'small' compared with the 'characteristic' length of $\Im$ (its diameter, for instance). If $e(s) = \varepsilon, \forall s \in \Im$, the shell is said to have a constant thickness of 2ε. If e is not a constant function, the shell is said to have a variable thickness.

The theory of elastic plates and shells is one of the most important theories of elasticity. Thin shells and plates are widely used in civil engineering projects as well as engineering projects. Examples include aircraft, cars, missiles, orbital launch systems, rockets, and trains.

Considerable work on the subject was conducted by the Russian scholars A.I. Lurje (1937), V.Z. Vlasov (1944) and V.V. Novozhilov (1951) after the pioneer idea of Love. However, now it appears necessary to improve the mathematical understanding of the classical plate and shell models pioneered by these scholars. The reason for this is that the precision required in aircraft and spacecraft projects has intensified with the advent of powerful electronic computers. The goal therein is, on the one hand, to develop better finite approximations on elements and, on the other hand, to refine theoretical models when necessary.

A.L. Gol'denveizer (1953) first put forward the ideas of conducting an asymptotic analysis based on the thickness of a shell or plate. New formulations for shells and plates were obtained by relaxing the constitutive relations and reinforcing the equilibrium that must be satisfied. Even in presenting a much more detailed analysis than that offered by his predecessors, no mathematical justification was given by Gol'denveizer. As such, a large number of difficulties were still to be overcome. At the same time, the weak formulations of J.L. Lions and mixed formulations of Hellinger-Reissner and Hu-Washizu (1968) appeared to provide alleviation. Of particular significance was the work of J.L. Lions (1973) on singular perturbation, which provided the tools and explanation for what happens when the asymptotic method is applied to plates, shells, and beams. The mathematical foundation of elasticity can be found in [4,6]. The asymptotic method was revisited in a functional framework proposed by Li, Zhang and Huang [1], P.G. Ciarlet [3,4] and M. Bernadou [5]. Their work made convergence and error analysis possible therein for the first time.

The 3D models are derived directly from the principles of equilibrium in classical mechanics, and are viewed as singular perturbation problems dependent upon the small parameter ε (the half-thickness of the shell). 2D models are obtained by making some additional hypotheses that are not justified by physical law. Our aim here is to justify mathematically the assumptions that formulate the basis of a 2D model of elasticity, and whose solution approaches 3D displacement better than the solution of classical models.

This paper is organized as follows: in Section 2, we present a mixed tensor analysis on 2D manifolds embedded into 3D Euclidian space; in Section 3, we provide the exchange of tensors and curvature after the deformation of curvatures; in Section 4, we give dif-