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Dynamics of a Predator-prey Model with Delay and Fear Effect*

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Abstract Recent manipulations on vertebrates showed that the fear of predators, caused by prey after they perceived predation risk, could reduce the prey's reproduction greatly. And it's known that predator-prey systems with fear effect exhibit very rich dynamics. On the other hand, incorporating the time delay into predator-prey models could also induce instability and oscillations via Hopf bifurcation. In this paper, we are interested in studying the combined effects of the fear effect and time delay on the dynamics of the classic Lotka-Volterra predator-prey model. It's shown that the time delay can cause the stable equilibrium to become unstable, while the fear effect has a stabilizing effect on the equilibrium. In particular, the model loses stability when the delay varies and then regains its stability when the fear effect is stronger. At last, by using the normal form theory and center manifold argument, we derive explicit formulas which determine the stability and direction of periodic solutions bifurcating from Hopf bifurcation. Numerical simulations are carried to explain the mathematical conclusions.

Keywords Predator-prey interaction, fear effect, delay, combined effect, Hopf bifurcation.

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1. Introduction

Predator-prey interactions play a crucial role in mathematical modeling of ecological processes. Following the work of Lotka and Volterra , there are extensive papers studying the mechanisms of predator-prey systems by observing direct killing of prey by predators. However, from about the early 1990s onwards, many theoretical biologists have argued that indirect effects, caused by costs of prey's behavioral defenses to perceived predation risk, could also alter the prey's reproductive physiology and demography powerfully (see [1, 2, 13]).

Animals will take many kinds of measures, including making fewer forays, enhancing vigilance, and even changing habitats, to cope with the perceived predation risk (for more details, we refer to [2, 17, 18] and [7]). Nevertheless, these antipredator responses could also cause harmful impacts on them. For example, when scared parents forage less, the birth rate is decreased and it will be more tough for the juvenile to survive because of starvation(see, e.g., [1–3]). Similarly, if prey

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migrate from the high-risk habitat to low-risk habitats to escape from predating, they may consume much energy especially when the environment of low-risk habitats are harsh(see [2, 16]). In general, such an anti-predator behavior can lower the reproductive rate in long term even though it enhances the probability of survival temporarily.

Except the viewpoints of these theoretical ecologists and evolutionary biologists, direct experimental evidence demonstrating that fear can affect the population of prey was given in 2011. In this year, Zanette et al. conducted an experimental study on wild, free-living song sparrows throughout their breeding season to test whether perceived predation risk could alone affect the number of offspring produced every year. And we can refer to the monograph [25] to obtain the specific experimental details. In this manipulation, females showed a variety of anti-predator responses, such as stopping incubation, foraging less and bringing less food to the nest. Several correlative experimental findings in [4–6, 9, 10, 20, 24] and [19] also suggested that fear of predators could alter prey's demography.

Based on the experimental facts in [27], Wang et al [25] considered a predator-prey model incorporating the cost of the fear into prey's reproduction. Their model is as follows

$$\begin{cases} u'(t) = ur_0 f(k,v) - du - au^2 - g(u)v, \\ v'(t) = v(-m + cg(u)), \end{cases}$$

where $u(t)$ and $v(t)$ denote the densities of prey and predator at time t, respectively, all the parameters are positive, r_0 is the birth rate of prey while d represents the natural death rate of prey, a describes the effect of intra-species competition of prey, c is the conversion rate of prey's biomass to predator's biomass, m is the natural death rate of predator, $g(u(t))$ represents the functional response between prey and predator, $f(k,v)$ accounts for the cost of anti-predator defense due to perceived predation risk, and k reflects the level of fear which drives anti-predator behaviors of the prey. By the biological meaning of k, v and $f(k,v)$, they assume that

$$\begin{cases} f(0,v) = 1, \qquad f(k,0) = 1, \quad \lim\limits_{k\to+\infty} f(k,v) = 0, \quad \lim\limits_{v\to+\infty} f(k,v) = 0, \\ \dfrac{\partial f(k,v)}{\partial k} < 0, \quad \dfrac{\partial f(k,v)}{\partial v} < 0. \end{cases}$$

Their mathematical analysis shows that high levels of fear can stabilize the predator-prey system by excluding the existence of periodic solutions. In addition, relatively low levels of fear can induce periodic solutions via Hopf bifurcation.

However, the model mentioned above only exhibits the change of all populations under ideal conditions. The authors assume that the predator population can convert the consumption into its growth instantaneously, which is obviously not so. In the dynamics of real populations, there are reaction-time lags in the response of predators(see,e.g. [21]), which appear as delays in the numerical response functions. Besides, it has been shown that delay differential equations exhibit much more complicated dynamics than ordinary differential equations in general since a time delay could cause a stable equilibrium to become unstable and cause the populations to fluctuate.

Thus, in this paper, by supposing $f(k,v) = \frac{1}{1+kv}$ and $g(u) = pu$ for the convenience of analysis, we incorporate a constant time delay τ into the above model in