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Oscillation Results for BVPs of Even Order Nonlinear Neutral Partial Differential Equations*

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Abstract A class of boundary value problems (BVPs) of even order neutral partial functional differential equations with continuous distribution delay and nonlinear diffusion term are studied. By applying the integral average and Riccati's method, the high-dimensional oscillatory problems are changed into the one-dimensional ones, and some new sufficient conditions are obtained for oscillation of all solutions of such boundary value problems under first boundary condition. The results generalize and improve some results of the latest literature.

Keywords Even order partial functional differential equation, boundary value problem, oscillation criteria, continuous distribution delay, nonlinear diffusion term.

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1. Introduction

The oscillation study of partial functional differential equations (PFDE) are of both theoretical and practical interest. Some applicable examples in such fields as population kinetics, chemistry reactors and control system can be found in the monograph of Wu [9]. There have been some results on the oscillations of solutions of various types of PFDE. Here, we mention the literatures of Kiguradze, Kusano and Yoshida [2], Thandapani and Savithri [8], Saker [5], Li and Debnath [3], Wang and Wu [10], Yang [12], Wang, Wu and Caccetta [11], ShouKaKu [6], ShouKaKu, Stavroulakis and Yoshida [7] and the references cited therein. To the best of our knowledge, there are fewer to investigate the oscillation of solutions of PFDE with continuous distribution delay. However, we note that in many areas of their actual

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application, models describing these problems are often effected by such factors as seasonal changes. Therefore it is necessary, either theoretically or practically, to study a type of PFDE in a more general sense–PFDE with continuous distribution delay. In this paper, we will discuss the oscillation of solutions of the high-order neutral partial functional differential equations with continuous distribution delay and nonlinear diffusion term

$$\begin{aligned}&\frac{\partial^n}{\partial t^n}[u+\int_c^d p(t,\eta)u[x,r(t,\eta)]d\tau(\eta)]+\int_a^b f(x,t,\xi,u[x,g(t,\xi)])d\mu(\xi)\\&=a_0(t)h_0(u)\Delta u+a_1(t)h_1(u(x,\sigma(t)))\Delta u(x,\sigma(t)),\quad (t,x)\in\Omega\times R_+\equiv G,\quad (1.1)\end{aligned}$$

where $n\geq 2$ is even, Ω is a bounded domain in R^m with a piecewise smooth boundary $\partial\Omega$, Δ is the Laplacian in R^m, $R_+=[0,\infty)$, the integral of Eq.(1.1) are Stieltjes ones.

Consider first boundary condition:

$$u(x,t)=0,\quad (x,t)\in\partial\Omega\times R_+. \qquad (1.2)$$

Throughout this paper, assume that the following conditions hold:

(H_1) $p(t,\eta)\in C(I\times[c,d],R)$, $I=[t_0,\infty)$, $t_0\in R$, $p(t,\eta)\geq 0$, $P(t)=\int_c^d p(t,\eta)d\tau(\eta)\leq P<1$, P is a constant;

(H_2) $r(t,\eta)\in C(I\times[c,d],R)$, $r(t,\eta)\leq t$, $\lim_{t\to\infty}\min_{\eta\in[c,d]}r(t,\eta)=\infty$;

(H_3) $g(t,\xi)\in C(I\times[a,b],R)$ is nondecreasing with respect to t and ξ, respectively, $\frac{d}{dt}g(t,a)$ exists, $g(t,\xi)\leq t$ for $\xi\in[a,b]$, $\lim_{t\to\infty}\min_{\xi\in[a,b]}g(t,\xi)=\infty$;

(H_4) $a_0(t),\ a_1(t)\in C(I,R_+)$, $\sigma(t)\in C(I,R)$, $\lim_{t\to\infty}\sigma(t)=\infty$;

(H_5) $h_0(u),\ h_1(u)\in C^1(R,R)$, $uh_0'(u)\geq 0$, $uh_1'(u)\geq 0$, $h_0(0)=0$, $h_1(0)=0$;

(H_6) $f(x,t,\xi,u)\in C(\Omega\times R_+\times[a,b]\times R_+,R)$;

(H_7) $\tau(\eta)$, $\mu(\xi)$ is nondecreasing on $[c,d]$ and $[a,b]$, respectively.

Definition 1.1. A function $u(x,t)\in C^n(G)\cap C^1(\overline{G})$ is said to be a solution of the boundary value problems (1.1), (1.2) if it satisfies (1.1) in G and boundary condition (1.2) in $\partial\Omega\times R_+$.

Definition 1.2. A solution $u(x,t)$ of the boundary value problems (1.1), (1.2) is said to be oscillatory in G if it has arbitrarily large zeros, namely, for any $T>0$, there exists a point $(x_1,t_1)\in\Omega\times[T,\infty)$ such that the equality $u(x_1,t_1)=0$ holds. Otherwise, the solution $u(x,t)$ is called nonoscillatory in G.

The objective of this paper is to derive some new oscillatory criteria of solutions of the boundary value problems (1.1), (1.2). It should be noted that in the proof we do not use the results of Dirichlet's eigenvalue problem.

To prove the main results of this paper, we need the following lemmas.

Lemma 1.1 (Kiguradze [1]). *Let $y(t)\in C^n(I,R)$ be of constant sign, $y^{(n)}(t)\neq 0$ and $y^{(n)}(t)y(t)\leq 0$ on I, then*

(i) there exists a $t_1\geq t_0$, such that $y^{(i)}(t)(i=1,2,\cdots,n-1)$ is of constant sign on $[t_1,\infty)$;

(ii) there exists an integer $l\in\{0,1,2,\cdots,n-1\}$, with $n+l$ odd, such that

$$y^{(i)}(t)>0,\ t\geq t_1,\ i=0,1,2,\cdots,l;$$

$$(-1)^{i+l}y^{(i)}(t)>0,\ t\geq t_1,\ i=l+1,\cdots,n.$$

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