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The Influence of Human Behavior on Heroin Dynamics

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Abstract In this paper, considering the influence of human behavior on heroin abuse, we establish a mathematical model to describe the spread of heroin. When the basic reproduction number is less than one, the heroin-free abuse equilibrium point is globally asymptotically stable. When the basic reproduction number is greater than one, the model has a unique heroin abuse equilibrium point which is globally asymptotically stable, and the heroin-free abuse equilibrium point is unstable. Finally, based on the partial rank correlation coefficients (PRCCs) and numerical simulations, the dynamic behavior of the model is further revealed. Our results show that human behavior can reduce the heroin abuse level.

Keywords Nonlinear incidence rate, Heroin model, Human behavior, Global asymptotic stability.

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1. Introduction

Heroin is an opioid drug made from morphine, a natural substance extracted from the seed pods of various opium poppy plants grown in Southeast and South-West Asia, Mexico and Colombia [1]. Heroin has become the most widely abused drug in the world. Among all drugs, drug crimes involving heroin manufacture, smuggling and abuse rank first. It is called the king of drugs in the world. Once one consumes heroin, the drug treatment can be divided into three stages: detoxification, rehabilitation and social return. Buprenorphine and methadone maintenance therapy are effective in the treatment of physical detoxification. Some studies have found that despite the relatively advanced conditions and methods for drug treatment, the relapse rate is still hovering 80%∼90% [2]. While social problems such as alcohol and drug use have been referred to in terms of epidemics, little has been published on the application of mathematical modelling methods to such problems [3]. Therefore, the problem of heroin abuse has attracted more and more attention of the society.

In the last decades, various mathematical modelling techniques have been extended for the purpose of understanding and combating heroin addiction prob-

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lems [3–13]. Using the idea of warehouse modeling in infectious disease dynamics, White and Comiskey applied infectious disease model to the study of drug users for the first time and established a kind of ordinary differential equation mathematical model of heroin drug transmission [3]. Subsequently, based on the literature [3], Samanta suggested that the coefficients of the model should depend on some long-term trends and seasonal changes of heroin epidemics. A non-autonomous heroin drug transmission model with distributed delays was established, and the sufficient conditions for the global asymptotic stability of the model were derived (see [4]). In [5], Fang and Li studied global asymptotic properties for an age-structured model of heroin use based on the principles of mathematical epidemiology where the incidence rate depends on the age of susceptible individuals. In [6], Wangari and Stone developed a model to test how heroin addiction spreads in society by using saturation functions to represent limited therapeutic effects and successful detoxification phenomena. The global stability of the model and an inherent backward bifurcation are obtained. In [14], Gao and Wang studied the impact of human behavior on cholera infection, local and global dynamics of the model are analyzed with respect to the basic reproduction number. Then they extend the ODE model to a reaction convection diffusion partial differential equation (PDE) model that accounts for the movement of both human hosts and bacteria. However, the influence of human behavior on heroin drug transmission and successful drug detoxification is rarely investigated in mathematical models.

To better describe the dynamic behavior of heroin drug transmission, in this paper, we consider the influence of human behavior on the transmission of heroin drugs and the successful detoxification of heroin addicts. This paper is organized as follows: In Section 1, we will establish a heroin abuse model. In Section 2, we will prove the local stability and global asymptotic stability of the equilibrium point. In Section 3, we will do numerical simulations and analyze the influence of parameters on the model. Finally, we will make a summary for this article.

2. Model description

The total population is divided into three compartments: heroin susceptible individuals, untreated heroin addicts, and treated heroin addicts, whose population size at time t was recorded as $S(t)$, $U_1(t)$, $U_2(t)$. The model is established as follows:

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = \Lambda - \beta(U_1) SU_1 - \mu S + \varepsilon U_1 + \eta U_2, \\ \frac{\mathrm{d}U_1}{\mathrm{d}t} = \beta(U_1) SU_1 + kU_2 - (p + \mu + \varepsilon + \delta_1) U_1, \\ \frac{\mathrm{d}U_2}{\mathrm{d}t} = pU_1 - (\mu + k + \eta + \delta_2) U_2. \end{cases} \tag{2.1}$$

In (2.1), the total population satisfies $N(t) = S(t) + U_1(t) + U_2(t)$. Λ is the number of susceptible individuals to heroin from the general population. k is the proportion of heroin addicts who relapse after cessation of treatment. η is the proportion of successful detoxification among heroin addicts receiving treatment. According to biological meaning we know $k > \eta$, because the rate of heroin relapse is higher than the rate of successful detoxification. p is the proportion of heroin addicts receiving treatment in detoxification centers. ε is the proportion of self-healing