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The Hopf Bifurcations in the Permanent Magnet Synchronous Motors*

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Abstract Based on the focus quantities and other techniques, the stability properties of equilibria and the limit cycles arising from Hopf bifurcations are investigated for two models of permanent magnet synchronous motors. The first model is of surface-magnet type and can have at most two unstable small limit cycles, which are symmetric with respect to x-axis. The other model is of interior-magnet type and can have at most four small limit cycles in two symmetric nests.

Keywords Focus quantity, Limit cycle, Hopf bifurcation.

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1. The mathematical model of PMSM

A motor is an electrical machine that converts electrical energy into mechanical energy. The permanent magnet synchronous motors (PMSM) are widely used in industry and electric vehicle applications. It has many advantages, such as high efficiency, high-power density and low-cost maintenance, see [2, 7] and references therein. There are two major types of PMSM: one with permanent magnets mounted on the rotor surface, called the surface-magnet type; and one with permanent magnets buried inside the rotor, called the interior-magnet type.

In [2], using the d-q frame, the PMSM model is written as

$$\begin{cases} L_{ds}\dfrac{i_{ds}}{dt'} = -u_{ds} - R_s i_{ds} + n_p\omega_r L_{qs} i_{qs}, \\ L_{qs}\dfrac{di_{qs}}{dt'} = -u_{qs} - R_s i_{qs} - n_p\omega_r L_{ds} i_{ds} + n_p\psi_a\omega_r, \\ J\dfrac{d\omega_r}{dt'} = T_m - \dfrac{3}{2}n_p\psi_a i_{qs} + \dfrac{3}{2}n_p(L_{ds} - L_{qs}) + i_{ds}i_{qs} - B_m\omega_r. \end{cases} \tag{1.1}$$

The variables and parameters are listed in Table 1.

For brevity, a symmetric load of resistance R is used, so that u_{ds} and u_{qs} can be expressed in terms of $i_{ds}R$ and $i_{qs}R$, respectively. Additionally, the net driving torque is considered to be proportional to i_{qs}, i.e., $T_m - T_{pm} = i_{qs}\mu$, where $T_{pm} = \dfrac{3}{2}n_p\psi_a i_{qs}$ is the PM torque and μ is a positive constant.

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Table 1. Variables and parameters of PMSM

Names	Descriptions	Units
i_{ds}, i_{qs}	Stator currents	A
u_{ds}, u_{qs}	Stator voltages	V
L_{ds}, L_{qs}	Stator inductances	H
R_s	Stator resistance	Ω
ψ_a	PM flux	Wb
n_p	Number of pole pairs	\
ω_r	Mechanical rotor speed	rad/s
T_m	Mechanical driving torque	N·m
J	Rotor inertia	kg·m^2
B_m	Viscosity friction coefficient	N·m·s

System (1.1) can be further simplified by transforming t' to τt, and i_{ds} to bkx, i_{qs} to ky, and ω_r to $\dfrac{z}{\tau n_p}$, where $b = \dfrac{L_{qs}}{L_{ds}}$, $\tau = \dfrac{L_{qs}}{R_s + R}$, and k is a positive constant. Therefore, system (1.1) can be rewritten as

$$\begin{cases} \dfrac{dx}{dt} = -bx + yz, \\ \dfrac{dy}{dt} = -y - xz + cz, \\ \dfrac{dz}{dt} = a(\gamma k y - z) + \eta k^2 xy, \end{cases} \tag{1.2}$$

where $a = \dfrac{B_m \tau}{J}$, $c = \dfrac{\psi_a}{k L_{qs}}, \eta = \dfrac{3n_p^2(L_{ds} - L_{qs}) b \tau^2}{2J}$, and $\gamma = \dfrac{n_p \tau \mu}{B_m}$.

Magnetic saliency describes the relationship between the rotor's flux (d-axis) inductance and the torque-producing (q-axis) inductance. Since surface-magnet PMSM exhibits no saliency (i.e. $L_{ds} = L_{qs}$), we have $\eta = 0$. In order to avoid the trivial case when $\gamma = 0$, γ is assumed nonzero and k is defined as $k = \dfrac{1}{\gamma}$. Thus, system (1.2) can be written as

$$\begin{cases} \dfrac{dx}{dt} = - x + yz, \\ \dfrac{dy}{dt} = - y - xz + cz, \\ \dfrac{dz}{dt} = a(y - z). \end{cases} \tag{1.3}$$

For more details, see [2].