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Modelling the *Wolbachia* Strains for Dengue Fever Virus Control in the Presence of Seasonal Fluctuation*

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Abstract Consider that infection with *Wolbachiacan* limit a mosquito's a-bility to transmit Dengue fever virus through its saliva, a mathematical model describing the transmission of Dengue fever between vector mosquitoes and human, incorporating *Wolbachia*-carrying mosquito population and seasonal fluctuation, is proposed. Firstly, the stability and bifurcation of this model are investigated exactly in the case where seasonality can be neglected. Further, the basic reproductive number $\mathcal{R}_0^s$ for this model with seasonal variation is obtained, that is, if $\mathcal{R}_0^s$ is less than unity the disease is extinct and $\mathcal{R}_0^s$ is greater than unity the disease is uniformly persistent. Finally, numerical simulations verify the theoretical results. Theoretical results suggest that, compared with the mosquito reduction strategies (such as the elimination of mosquito breeding sites, killing of adult mosquitoes by spraying), introducing *Wolbachia* strains is as effectual to fight against the transmission of Dengue virus.

Keywords Dengue fever, *Wolbachia*, Seasonal fluctuation, Stability and sensitivity analysis, Extinction and persistence.

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1. Introduction

Dengue fever is a viral disease mainly prevalent in tropical and subtropical regions of the world, which is transmitted by the bite of an *Aedes* mosquito infected with Dengue virus. It is estimated that approximately 1.5 billion people are at risk, and may be 50-100 million individuals are affected by Dengue fever virus each year [23]. Currently, there are still no specific antiviral therapy or vaccines available to combat Dengue fever [6], so the method of controlling the vector population is still a main measure to prevent the transmission of Dengue fever virus. It is well-known that traditional measures such as the use of insecticides to reduce the mosquito population tend to be very expensive, unsustainable, environmentally undesirable, which may indeed lead to insecticide resistance [6].

Experiments and field trials have demonstrated that the intracellular bacterium

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Wolbachia is a maternally transmitted endosymbiotic bacterium that is estimated to infect as much as 65% of insect species and have been surveyed that infect about 28% of mosquito species [9, 22]. It lives in the testes and ovaries of hosts and interferes with the reproductive mechanisms, inducting a variety of mosquito phenotypes such as those with cytoplasmic incompatibility (CI), parthenogenesis, feminization of genetic males and so on. The effects of CI causes embryos from females uninfected with *Wolbachia* to die when they are mated with infected males. Whereas, infected females are not affected in this manner [11]. In mosquitoes vectors, *Wolbachia* induced CI and matrilinear inheritance may have opposite effects, such as population extinction, coexistence or all of the uninfected population may be replaced by infected insects. Some *Wolbachia* cannot be only successfully spread within mosquito populations through CI, but also prevent mosquito host to replicate and spread Dengue fever virus [10, 20].

Recently, McMeniman et al. [13] have proposed a *Wolbachia* infection in an *Aedes aegypti* population through microinjecting bacteria into the mosquito embryos and reducing the transmission of Dengue fever virus from mosquitoes to humans. New analysis of that data shows that there are usually two ways in which *Wolbachia*-infected mosquitoes may be inferior Dengue vectors, one reduces adult survival sufficiently may result in very few infected mosquitoes reaching the infectious stage [18], another limits a mosquito's ability to transmit Dengue fever virus through its saliva [10]. Taking these means into account, there are many mathematical models (including discrete-time and continuous-time models) are investigated for the effects of *Wolbachia* infection (see, e.g., [2,16,25] and the references therein).

On the other hand, for infectious diseases spreading by vectors, the seasonality is apparent on the varying contact rate over the years [3]. It becomes natural to model these diseases as periodically forced nonlinear systems [8]. The seasonality is often seen as the main factor responsible for periodic epidemic cycles, and different kinds of seasonality sources are analyzed. For example, varying transmission rates [4], the volatility of birth rates [12], vaccination program [14] and so on. Particularly, the increased intensity and frequency of El Nio in the past few years, which leads to frequent outbreaks and quick spread of insect-borne infectious diseases around the world. Therefore, how to control and eliminate vector-borne diseases should be one of worldwide public health problems.

Based on the above discussion, we propose a mathematical model of the transmission dynamics of Dengue fever in vector mosquitoes and human population in this paper, where *Wolbachia*-carrying mosquito population and seasonal fluctuation are introduced. The main purpose is to discuss the effects of *Wolbachia* and seasonal change for the control and elimination of Dengue fever virus. The rest of the paper is outlined as follows: a basic mathematical model with *Wolbachia* and seasonal fluctuation for Dengue virus transmission dynamics and some preliminaries are proposed in Section 2. We consider the existence and stability of equilibria in Section 3. In Section 5, we investigate the extinction and uniform persistence of Dengue fever in the presence of seasonal fluctuation. Section 6 contains numerical simulations for theoretical results and biological conclusions.

2. Model formulation and preliminaries

We set up a mathematical model to study how introducing *Wolbachia* into a mosquito population might affect the transmission of Dengue fever virus. In it,