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Lump Solutions, Lump-soliton Interaction Phenomena and Breather-soliton Solutions to a (3+1)-dimensional Boiti-Leon-Manna-Pempinelli-like Equation*

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Abstract The (3+1)-dimensional Boiti-Leon-Manna-Pempinelli-like equation (BLMP-like equation) is introduced by the generalized bilinear operators D_p associated with $p = 3$. The lump solutions, lump-soliton interaction phenomena and breather-soliton solutions are discussed to the (3+1)-dimensional BLMP-like equation based on the generalized bilinear method with symbolic computation system Mathematica. In order to observe the behavior of those solutions, we fix the value of z, then give the 3D-graphs of some solutions at different times. We find a lump solution moved in oblique direction; a lump-soliton interaction phenomenon is appeared and disappeared along with the time. We also see a kink-breather soliton moved in oblique direction.

Keywords Interaction phenomena, Lump solution, (3+1)-dimensional Boiti-Leon-Manna-Pempinelli-like equation.

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1. Introduction

Many physical and mechanical phenomena are nonlinear. In this way, solving nonlinear differential equations has become a hot issue in the field of applied mathematics. So far, many researchers have proposed and developed various techniques for solving nonlinear differential equations, such as Lie symmetry [3, 35], Hirota bilinear method [4, 12, 20, 21, 25, 30], homotopy perturbation method [10, 11, 33, 34], homotopy analysis method [13], the Adomian decomposition method [1, 27] (ADM), auxiliary equation methods [8, 32] and so on.

Among those techniques, the Hirota bilinear method was proposed by Japan's famous mathematician and physicist Ryogo Hirota [12] for solving nonlinear differential equations. If a nonlinear differential equation has the bilinear form, one can find its many physical characteristics by its many kinds of exact solutions. Based on the Hirota bilinear operator, in recent years, researchers found lump solutions of some differential equations to explain many physical phenomena in various subjects, including plasma physics, shallow water waves and optical media [9, 26, 28].

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Based on Hirota bilinear operators, Ma [17–19] proposed the generalized bilinear differential operators as follows:

$$
\begin{aligned}
& D_{p,x}^{m} D_{p,t}^{n} f \cdot f \\
&= \left(\frac{\partial}{\partial x} + \alpha_p \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} + \alpha_p \frac{\partial}{\partial t'}\right)^n f(x,t) f(x',t') \Bigg|_{\substack{x'=x \\ t'=t}} \\
&= \sum_{i=0}^{m} \sum_{j=0}^{n} \binom{m}{i} \binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m-i}}{\partial x^{m-i}} \frac{\partial^{i}}{\partial x'^{i}} \frac{\partial^{n-j}}{\partial t^{n-j}} \frac{\partial^{j}}{\partial t'^{j}} f(x,t) f(x',t') \Bigg|_{\substack{x'=x \\ t'=t}} \\
&= \sum_{i=0}^{m} \sum_{j=0}^{n} \binom{m}{i} \binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m+n-i-j} f(x,t)}{\partial x^{m-i} t^{n-j}} \frac{\partial^{i+j} f(x,t)}{\partial x^i t^j}, m, n \geq 0, \qquad (1.1)
\end{aligned}
$$

where the α_p^s satisfies:

$$\alpha_p^s = (-1)^{r_p(s)}, \quad s = r_p(s) \bmod p\,,$$

and

$$\alpha_p^i \alpha_p^j \neq \alpha_p^{i+j}, \quad i, j \geq 0\,,$$

when the prime number $p \geq 2$. For different prime numbers p, Hirota bilinear equations have been generalized to generate diverse nonlinear differential equations possessing potential applications. In recent years, based on the generalized bilinear differential operators, the lump solutions, rational solution and interaction solutions are discussed. For example, new periodic wave, cross-kink wave and the interaction phenomenon for the Jimbo-Miwa-like equation [36], lump solutions with higher-order rational dispersion to the second KPI equation [23], lump and interaction solutions to linear PDEs in 2+1 dimensions [22], determining lump solutions for a combined soliton equation in (2+1)-dimensions [29], lump solutions to a combined equation involving three types of nonlinear terms relations [24], lump solutions to a (2+1)-dimensional fifth-order KdV-like equation [2], lump solutions to dimensionally reduced Kadomtsev-Petviashvili-like equations [31], interaction solutions to the (3+1)-dimensional Kadomtsev-Petviashvili-Boussinesq-like equation [15], rational solutions to two Sawada Kotera-like equations [6], rational solutions to a Hirota-Satsuma-like equation [14], rational solutions to an extended Kadomtsev-Petviashvili-like equation [16], rational solutions to a Kdv-like Equation [37] and rational solutions to a generalized (2+1)-dimensional Shallow-Water-Wave-like equation [7].

In this paper, based on the above techniques, we would like to introduce a BLMP-like equation by the generalized bilinear operators. Then, we will discuss its lump solutions, lump-soliton interaction phenomena and breather-soliton solutions.

2. The (3+1)-dimensional BLMP-like equation

The (3+1)-dimensional BLMP equation [5] is as follows:

$$u_{yt} + u_{zt} + u_{xxxy} + u_{xxxz} - 3u_x(u_{xy} + uxz) - 3u_{xx}(u_y + u_z) = 0.$$