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Travelling Wave Solutions and Conservation Laws of the (2+1)-dimensional Broer-Kaup-Kupershmidt Equation*

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Abstract The travelling wave solutions and conservation laws of the (2+1)-dimensional Broer-Kaup-Kupershmidt (BKK) equation are considered in this paper. Under the travelling wave frame, the BKK equation is transformed to a system of ordinary differential equations (ODEs) with two dependent variables. Therefore, it happens that one dependent variable u can be decoupled into a second order ODE that corresponds to a Hamiltonian planar dynamical system involving three arbitrary constants. By using the bifurcation analysis, we obtain the bounded travelling wave solutions u, which include the kink, anti-kink and periodic wave solutions. Finally, the conservation laws of the BBK equation are derived by employing the multiplier approach.

Keywords The (2+1)-dimensional Broer-Kaup-Kupershmidt equation, Travelling wave solutions, Conservation laws, Multiplier method.

1. Introduction

A significant amount of mathematical research has been dedicated to developing tools for the treatment of nonlinear partial differential equations (NLPDEs). This is necessitated by the fact that NLPDEs have diverse applications in the physical world and their solutions help to shed light on the various phenomena with which we interact. The research has been in part vested in developing methods of obtaining their exact solutions. Here, we give a few of these methods: the Lie symmetry method [5, 18, 19], the inverse scattering transform method [1], the tanh-function method and extended tanh-function method [17], Symbolic methods [9], the Riccati equation method [16], the Jacobi elliptic function method [20], the exp-function method [8], the homogeneous balance method [23], Hirota's bilinear method [10], simple transformation method [13], F-expansion method [26], dynamical system method [27] and so on.

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The Broer-Kaup-Kupershmidt (BKK) system of equations

$$\begin{aligned} &u_{ty} - u_{xxy} + 2(uu_x)_y + 2v_{xx} = 0, \\ &v_t + v_{xx} + 2(uv)_x = 0. \end{aligned} \tag{1.1}$$

is one of the most popular system of NLPDEs to emerge in the past few decades. This is evidenced by the vast array of scholars who have researched various aspects of the system. It has applications in fluid dynamics where it models dispersive shallow water waves travelling in equal depth. By using a convenient scaling transformation, it has been shown in [6, 24] that the (2+1)-dimensional asymmetric Davey-Stewartson system [15]

$$\bar{q}_t + \frac{\bar{q}_{xx}}{2} + 2\bar{q}\partial_y^{-1}(\bar{q}\bar{r})_x = 0, \quad \bar{r}_t + \frac{\bar{r}_{xx}}{2} + 2\bar{r}\partial_y^{-1}(\bar{q}\bar{r})_x = 0,$$

transforms into (1) under the transformation

$$\bar{q} = \exp\left(-\int^x u\mathrm{dx}\right), \quad \bar{r} = -v\exp\left(\int^x u\mathrm{dx}\right).$$

Solitoff and dromion solutions are obtained in the same work [6]. Also, the Kadomtsev-Petviashvili equation transforms into the BKK equation under a symmetry constraint, see for example [15, 26]. In [26], the modified extended Fan sub-equation method is used to obtain soliton-like and Jacobi elliptic wave function-like solutions of (1). In [14], Bäcklund transformation and variable separation approach was used to obtain dromions, lumps and peakons through the introduction of an arbitrary function. Again, in [22, 25] an auxiliary equation method was utilised to obtain its exact travelling wave solutions. His semi-inverse method was applied in [28] to establish a variational principle of the BKK system. Several other researchers have utilised different ad hoc methods to establish different solutions of (1). Kassem and Rashed [11] came up with closed form solutions of (1) by using hidden symmetries of its Lie optimal systems. The most recent work on the BKK system was by Tang et al. [21] who presented the double Wronskian solutions by using Hirota's method and binary bell polynomials.

Conservation laws depict conserved quantities of physical interest. The most common physical quantities that are conserved are energy, charge, momentum and mass amongst others. Conservation laws also help in establishing the uniqueness, stability and existence of solutions of differential equations. There are several methods available for deriving conserved quantities, see for example [2–4, 7, 12].

Due to its undeniably vast applicability, continued study of the BKK system remains necessary. In this paper, we further explore the (2+1)-dimensional BKK system (1). Unlike most of the previous research on this NLPDE, we do not employ ad hoc methods to obtain its analytic solutions, but utilise a standard Lie based integration method [18]. Here, we provide a detailed outline of the derivation of the bounded travelling wave solutions, which include kink and anti-kink profiles. We also outline how periodic solutions of a snoidal nature are obtained. Moreover, the homotopy integral approach to finding conservation laws is explored in details. To the best of our knowledge, the literature is devoid of explicit applications of this approach, moreso for nonlinear partial differential equations with mixed derivatives, our work is novel in this regard.