Journal of Nonlinear Modeling and Analysis http://jnma.ca; http://jnma.ijournal.cn
Volume 3, Number 4, December 2021, 505–521 DOI:10.12150/jnma.2021.505

An Analogue-difference Method and Application to Induction Motor Models*

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Abstract The paper established a so-called analogue-difference method (AD-M) to compute the numerical solutions for boundary value problems of higher-order differential equations, which can be a fundamental method and performs much better than the finite difference method (FDM), even for second-order boundary value problems. Numerical examples and results illustrate the simplicity, efficiency and applicability of the method, which also show that the proposed method has obvious advantages over the methods presented by recent state-of-the-art work for induction motor models.

Keywords Difference method, Analogue-difference method, Numerical solution.

MSC(2010) 65LXX, 34K15.

1. Introduction

The difference method (FDM) is a fundamental method to find numerical solutions for boundary value problems of ordinary differential equations. Since the simplicity and effectiveness, FDM has been applied to solve numerical solutions for second-order boundary value problems of ordinary differential equations and partial differential equations (see [3]- [11]), and this method can be found in many text books and papers related to numerical methods, see [2], [4], [7], [11] and references therein. In particular, [4] present the difference method for the classical second-order boundary value problem in details. However, this method is not satisfied, and it is even invalid in some situation for higher-order boundary value problems. On the other hand, biologically inspired intelligent computing approach, based on artificial neural networks (ANN) models, the authors of [1] established some methods by optimising efficient local search methods so-called sequential quadratic programming (SQP), interior point technique (IPT) and active set technique (AST), they applied

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*The authors were supported by National Natural Science Foundation of China (Nos. 11361047, 11561043) and the Natural Science Foundation of Qinghai Province (No. 2017-ZJ-908).

the methods to solve some fifth-order boundary value problems arisen in induction motor. The authors show that their proposed technique is state-of-the-art, which is good in accuracy. In [7] and [6], the authors developed the so-called new finite difference method to solve a second-order boundary value problem, and a eighth-order boundary value problem respectively. However, the methods are only fourth-order accurate. In this paper, we devote ourselves to improving the accuracy and efficiency of FDE by using the higher-order derivative substitution formulation based on Taylor expansion. Our new method can not only be applied to second-order boundary value problems, but also higher-order boundary value problems especially, and it also can be applied to all models of [1], which shows that the method has higher accuracy than those of [1].

To self completeness and clearness, we introduce some basic knowledge for FDM (see [4] and references therein). Consider the following second-order BVP

$$\begin{cases} Lu \equiv -u'' + q(t)u = f(t), a < t < b, \\ u(a) = \alpha, u(b) = \beta, \end{cases} \tag{1.1}$$

where $q(t), f(t) \in C[a,b], q(t) \geq 0$ for $t \in [a,b]$. Assumed that the problem (1.1) has a unique solution. The process of numerical solution using classical difference method as follows.

We divide the interval $[a,b]$ into N equal parts, and take the grid points as follows:

$$a = t_0 < t_1 < \cdots < t_i < \cdots < t_N = b, \tag{1.2}$$

where $t_i = a + ih$, step length $h = \frac{b-a}{N}$. Choose the second-order center difference quotient formula at node t_i

$$u''(t_i) \approx \frac{1}{h^2}[u(t_{i-1}) - 2u(t_i) + u(t_{i+1})], \tag{1.3}$$

then the following system obtained

$$\begin{cases} L_h u_i = -\frac{1}{h^2}(u_{i-1} - 2u_i + u_{i+1}) + q_i u_i = f_i, i = 1, 2, \cdots, N-1, \\ u_0 = \alpha, u_N = \beta, \end{cases} \tag{1.4}$$

where $q_i = q(t_i), f_i = f(t_i), i = 0, 1, \cdots, N$. The truncation error of this method is

$$R_i(u) = Lu(t_i) - L_h u(t_i) = \frac{h^2}{12} u^{(4)}(\xi_i), \quad \xi_i \in (t_{i-1}, t_{i+1}), \tag{1.5}$$

where L is the derivative operator defined by $Lu := u''$. The algebraic system (1.4) can be written in matrix form

$$\begin{pmatrix} -\frac{1}{h^2} & \frac{2}{h^2}+q_1 & -\frac{1}{h^2} & & & \\ & -\frac{1}{h^2} & \frac{2}{h^2}+q_2 & -\frac{1}{h^2} & & \\ & & \ddots & \ddots & \ddots & \\ & & & -\frac{1}{h^2} & \frac{2}{h^2}+q_{N-1} & -\frac{1}{h^2} \\ 1 & 0 & \cdots\cdots\cdots & & 0 & 0 \\ 0 & 0 & \cdots\cdots\cdots & & 0 & 1 \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \\ \vdots \\ \vdots \\ \vdots \\ u_N \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_{N-1} \\ \alpha \\ \beta \end{pmatrix}, \tag{1.6}$$