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Positive Solutions for a Stationary Prey-Predator Model with Density-Dependent Diffusion and Hunting Cooperation*

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Abstract This paper concerns a stationary prey-predator model with density-dependent diffusion and hunting cooperation under homogeneous Dirichlet boundary conditions. Based on the spectral analysis, the asymptotic stability of trivial and semi-trivial solutions is obtained. Moreover, the sufficient conditions for the existence of positive solutions are established by using degree theory in cones. Our analytical results suggest that density-dependent diffusion and hunting cooperation obviously influence on the positive solutions.

Keywords Prey-predator model, density-dependent diffusion, hunting cooperation, positive solutions

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1. Introduction

The present paper is concerned with the following Dirichlet problem of quasilinear elliptic equations:

$$\begin{cases} -d_u \Delta u = ru - u^2 - (1+\alpha v)uv, & x \in \Omega, \\ -\Delta\left[\left(d_v + \frac{\beta}{1+\gamma u}\right)v\right] = mv - v^2 + c(1+\alpha v)uv, & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, the parameters $d_u, d_v, \beta, \gamma, \alpha, r, c$ are positive constants and m may change sign. System (1.1) is the stationary problem of a prey-predator model in which unknown functions $u = u(x)$ and $v = v(x)$ denote the stationary population densities of the prey and the predator in the habitat Ω, respectively. In the reaction terms, r and m are the growth rates of respective species; α describes predator cooperation in hunting; c accounts for

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the intrinsic predation rate. In the diffusion terms, $d_u\Delta u$ and $d_v\Delta v$ denote the linear diffusion driven by the dispersive force associated with random movement of each species, while the nonlinear diffusion $\Delta\left(\frac{\beta v}{1+\gamma u}\right)$ describes a situation in which the predator chases the prey: β is the cross-diffusion pressures, and γ represents the interference rate from the prey in the chase by the predator. For more details on the backgrounds of density-dependent diffusion and hunting cooperation, we refer to [1] and [11].

When $\alpha = 0$ and $\gamma = 0$, (1.1) is reduced to the classical Lotka-Volterra prey-predator model which has received extensive study in the last decade (see [2,8,9,16] and references therein). When $\alpha = 0$ and $\gamma > 0$, K. Kuto and his collaborators established the existence of positive solutions by the bifurcation theory in [4,6] and discussed the limiting behavior of positive solutions in [5, 7]. However, as far as we know, there are few works on the positive solutions in the case where $\alpha > 0$ and $\gamma > 0$. It is worth noting that, although the literature on hunting cooperation is limited till now, some recent works can be found which address the effect of cooperative hunting [3, 12, 14, 15, 17] and the references therein.

The purpose of this paper is to establish the asymptotic stability of trivial and semi-trivial solutions and provide the sufficient conditions for the existence of positive solutions. To present our main result, we introduce some notations. For any given $d > 0$ and $q(x) \in C(\overline{\Omega})$, the eigenvalue problem

$$-d\Delta\phi + q(x)\phi = \lambda\phi, \quad x \in \Omega, \qquad \phi = 0, \quad x \in \partial\Omega$$

has an infinite sequence of eigenvalues denoted by $\{\lambda_i(d, q(x))\}_{i=1}^{\infty}$. Additionally, for any given $d > 0$, the logistic equation

$$-d\Delta\phi = a\phi - \phi^2, \quad x \in \Omega, \qquad \phi = 0, \quad x \in \partial\Omega$$

admits a unique positive solution if and only if $\lambda_1(d, -a) < 0$, which is denoted by $\theta_{d,a}$.

Our first theorem gives the asymptotic stability of trivial and semi-trivial solutions.

Theorem 1.1. *The following statements hold true.*

(1) Trivial solution $(0, 0)$ is asymptotically stable if $\lambda_1(d_u, -r) > 0$ and $\lambda_1(d_v + \beta, -m) > 0$, while it is unstable if $\lambda_1(d_u, -r) < 0$ or $\lambda_1(d_v + \beta, -m) < 0$.

(2) Assume that $\lambda_1(d_v + \beta, -m) < 0$. Then $(0, \theta_{d_v+\beta,m})$ is asymptotically stable if

$$\lambda_1(d_u, (1 + \alpha\theta_{d_v+\beta,m})\theta_{d_v+\beta,m} - r) > 0;$$

while it is unstable if

$$\lambda_1(d_u, (1 + \alpha\theta_{d_v+\beta,m})\theta_{d_v+\beta,m} - r) < 0.$$

(3) Assume that $\lambda_1(d_u, -r) < 0$. Then $(\theta_{d_u,r}, 0)$ is asymptotically stable if

$$\lambda_1\left(1, -\frac{(m + c\theta_{d_u,r})(1 + \gamma\theta_{d_u,r})}{d_v + d_v\gamma\theta_{d_u,r} + \beta}\right) > 0;$$

while it is unstable if

$$\lambda_1\left(1, -\frac{(m + c\theta_{d_u,r})(1 + \gamma\theta_{d_u,r})}{d_v + d_v\gamma\theta_{d_u,r} + \beta}\right) < 0.$$