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Unbiased Grey Polynomial Model Based on Precise Direct Integration Method*

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Abstract During the conversion from difference to differential in the grey polynomial model, the "misplaced replacement" problem will occur. A novel unbiased grey polynomial model, i.e., HUGMP$(1, 1, N)$ is presented to overcome the above drawback. Meanwhile, the parameter estimation of HUGMP $(1, 1, N)$ is directly constructed by the equivalent relation between the parameter estimation and the recurrence relation of the time response function for GMP$(1, 1, N)$ model. The recurrence relation is deduced from the solution of the homogenized differential equations, converted from the whitenization equation of GMP$(1, 1, N)$ model by introducing new variables. The simulated values are directly calculated by the precise direct integration method in order to reduce round-off error and improve fitting accuracy. Moreover, it is proved that the proposed unbiased grey polynomial model possesses not only complete coincidence of simulation to non-homogeneous exponential sequence with polynomial time terms, but also multiple transformation consistency. At last, the results of applications verify the effectiveness of the proposed model by comparing with other conventional models.

Keywords Grey system theory, time power, GMP$(1, 1, N)$ model, unbiased grey model, precise direct integration method

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1. Introduction

Since Professor Deng established grey system theory for small sample and poor information problems, grey forecasting model has been successfully applied in industry, agriculture, energy and so on [1, 2]. As a classical form of grey forecasting model, GM$(1, 1)$ model is proposed to fit exponential sequences, but it can't fully fit the sequence even if the original data completely conforms to the homogeneous exponential law, because there exists "misplaced replacement" between parameter estimation and time response of GM$(1, 1)$ model. Therefore unbiased GM$(1, 1)$ models are presented to eliminate "misplaced replacement" and simulate homogeneous exponential sequences without any bias [3–7], which improve the fitting precision. The study of unbiased grey models becomes an important issue for grey forecasting models. The unbiased grey models are usually constructed by using direct modeling

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method [3–5], grey derivatives optimization, background values optimization [6,7] and discrete grey models. Among these, direct modeling method is considered as the unity of grey derivative optimization and background value optimization [9], the key of which is to seek the equivalent relation of parameter estimation and time response function in order to overcome the inconsistence from difference to differential in grey models. Moreover, the above three methods don't change the basic form of $\mathrm{GM}(1,1)$ model, but the discrete grey model ($\mathrm{DGM}(1,1)$) is not the precise form of $\mathrm{GM}(1,1)$ model [14], where both the parameter estimation and predicting adopt the difference form. Here, we discuss the unbiased grey models based on direct modeling method for the base form of grey models.

Researchers improve the structure of $\mathrm{GM}(1,1)$ model to construct various grey models to fit different structural data, non-monotonic time series, S-type data, periodic time sequence and other nonlinear data. Then for grey models with simple model structure, non-homogeneous grey model $\mathrm{NGM}(1,1,k)$ model [8,9], Verhulst model [10–12] and grey Bernoulli model [13], their unbiased grey models also have been presented to overcome the "misplaced replacement" problem. The equivalent relations between parameter estimation and time response function of the Verhulst model and grey Bernoulli model are obtained after they are converted to $\mathrm{GM}(1,1)$ model by introducing transformations, and the relation of $\mathrm{NGM}(1,1,k)$ model is easily obtained because the grey action is a linear function. Correspondingly, constructing unbiased grey models for complex grey actions such as, grey Riccati model [15–17], fractional-order grey model [18–20], grey model based on trigonometric function [21], grey model with time power terms [22–24], and grey polynomial model [25–30], is challenging by using direct modeling methods.

Among the complex grey models, grey polynomial model ($\mathrm{GMP}(1,1,N)$) designed by introducing polynomial time terms to the grey action, can describe the original sequence in accordance with a more general trend rather than the special homogeneous or non-homogeneous trend [25]. In order to improve the fitting accuracy and expand its applications for various data, researchers have proposed many improved $\mathrm{GMP}(1,1,N)$ models. Liu et al. put forward extended $\mathrm{GMP}(1,1,N)$ models by introducing the fractional accumulating generation operator and fractional power time terms [26,27]. Li et al. improved $\mathrm{GMP}(1,1,N)$ model by constructing the tuned background coefficient using optimal methods [28]. Wei et al. gave the simulated value of $\mathrm{GMP}(1,1,N)$ model according to the connotation method instead of the whitenization method and proved that the model simulates non-homogeneous exponential sequences without error [29] and proposed a discrete grey polynomial model [30]. But the unbiased $\mathrm{GMP}(1,1,N)$ model based on direct modeling method has not been given yet. Especially, when $N=1$, $\mathrm{GMP}(1,1,N)$ model is simplified to $\mathrm{NGM}(1,1,k)$ model, and its unbiased model has been studied. Unfortunately, for $N>1$, the complexity of polynomial grey action makes it difficult to deduce the recurrence relation of the time response sequence, although the time response function of $\mathrm{GMP}(1,1,N)$ model is easily given. Accordingly, the equivalent relation of parameter estimation and time response sequence can't be built. Thus we propose a novel way to gain the recurrence relation of time response sequence of $\mathrm{GMP}(1,1,N)$ model. The whitenization equation of $\mathrm{GMP}(1,1,N)$ model is firstly transformed to a first order homogeneous differential equations by introducing new variables, then the recurrence relation is deduced by the solution of the homogenized differential equations by the exponential matrix form. Therefore, we obtain the equivalent relation between the parameter estimation formula and the