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Qualitative Analysis of a Predator-prey System with Ratio-dependent and Modified Leslie-Gower Functional Response*

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Abstract In this paper, a predator-prey model with ratio-dependent and modified Leslie-Gower functional response subject to homogeneous Neumann boundary condition is considered. First, properties of the constant positive stationary solution are shown, including the existence, nonexistence, multiplicity and stability. In addition, a comparatively characterization of the stability is obtained. Moreover, the existing result of global stability is improved. Finally, properties of nonconstant positive stationary solutions are further studied. By a priori estimate and the theory of Leray-Schauder degree, it is shown that nonconstant positive stationary solutions may exist when the system has two constant positive stationary solutions.

Keywords Predator-prey model, Modified Leslie-Gower functional response, Stability, Nonconstant positive stationary solution.

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1. Introduction

The investigations on the relationship between predator and prey are of fundamental importance in mathematical ecology, the predator-prey models have been extensively studied during recent years. More and more realistic models have been derived by virtue of laboratory experiments and observations since the Lotka-Volterra predator-prey model. In [13], Leslie emphasized that the growth rate of the prey and predator has an upper limit and proposed the following predator-prey model

$$\begin{cases} \dfrac{du}{dt} = u(a - bu) - p(u)v, \quad t > 0, \\ \dfrac{dv}{dt} = v\left(d - \dfrac{hv}{u}\right), \quad t > 0, \\ u(0) > 0, \quad v(0) > 0, \end{cases} \tag{1.1}$$

where u and v represent the species densities of prey and predator respectively. The term hv/u is usually called the Leslie-Gower functional response, which explains the loss of predator species due to the rarity of its favorite food u. The term $p(u)$ is the functional response of predator to prey. As $p(u)$ is Holling-II type, (1.1) has

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been shown to exhibit quite rich behaviors, such as stable limit cycles, semi-stable limit cycles, global stability of the unique constant positive stationary solution, bifurcation and periodic solutions, see [5,6,8,9,18,19,25].

Taking into account the inhomogeneous distribution of the species in different spatial locations, and the natural tendency of each species to diffuse to areas of smaller population concentration, (1.1) becomes the following system

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1 \Delta u = u(a - bu) - p(u)v, & x \in \Omega, t > 0, \\ \dfrac{\partial v}{\partial t} - d_2 \Delta v = v\left(d - \dfrac{hv}{u}\right), & x \in \Omega, t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x) > 0,\ v(x,0) = v_0(x) > 0,\ x \in \bar{\Omega}. \end{cases} \tag{1.2}$$

As $p(u) = \beta u$, Du and Hsu [7] obtained the global stability of the unique constant positive stationary solution and conjectured that the global stability is true without any restriction. So nontrivial spatial patterns may not be expected for (1.2) with $p(u) = \beta u$. As $p(u)$ is taken as $u/(m+u)$, Peng and Wang [21,22] analyzed the global stability of the unique constant stationary solution and deduced that nonconstant positive stationary solutions may exist for (1.2).

The functional response of predator to prey can be classified as prey-dependent and predator-dependent types. The prey-dependent functional response only involves the prey u, which means that the prey density alone determines the predation behavior o f the predator. However, some recent numerical examples from biological control reveal that the classical prey-dependent functional response can provide contrast to the realistic observations [24]. Moreover, there is growing biological and physiological evidence that in many cases, especially when predators have to search, share and compete for food, a more suitable functional response should be the so-called ratio-dependent one, which is predator-dependent [1–3]. For the predator-prey model with ratio-dependent functional response, one can refer to [12,15,27] and references therein. In particular, Peng and Wang [23] investigated the following predator-prey model with ratio-dependent functional response

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1 \Delta u = u(\lambda - u) - \dfrac{\beta uv}{u + mv}, & x \in \Omega, t > 0, \\ \dfrac{\partial v}{\partial t} - d_2 \Delta v = v\left(1 - \dfrac{v}{u}\right), & x \in \Omega, t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x) > 0,\ v(x,0) = v_0(x) > 0,\ x \in \bar{\Omega}, \end{cases} \tag{1.3}$$

where λ, m, β, d_1 and d_2 are positive constants. In (1.3), $p(u) = u/(u+mv)$ is just the ratio-dependent functional response, in which the parameters m and β account for the saturation rate and the predation rate of the predator, respectively. The Leslie-Gower functional response v/u is also considered in (1.3), while in the case of severe scarcity, predator can switch to other populations but its growth is limited by the fact that its most favorite food is not available in abundance. This situation can be taken care of by adding a positive constant to the denominator, that is, $v/(u+k)$ [4].

Motivated by the above work, this paper is concerned about the following predator-prey model with ratio-dependent and modified Leslie-Gower functional response:

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1\Delta u = u(\lambda - u) - \dfrac{\beta uv}{u+mv}, & x \in \Omega, t > 0, \\ \dfrac{\partial v}{\partial t} - d_2\Delta v = v\left(1 - \dfrac{v}{u+k}\right), & x \in \Omega, t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x) > 0,\ v(x,0) = v_0(x) > 0,\ x \in \bar{\Omega}. \end{cases} \tag{1.4}$$

Here, k is a positive constant and represents the largest carrying capacity of the predator when the predator population lacks its favorite food. In [29], Zhao and Bie investigated the persistence of system (1.4). When the constant positive stationary solution is unique, they used the method of upper-lower solutions to obtain the global stability of the constant positive stationary solution under some special conditions. Whereas, (1.4) may exhibit two constant positive stationary solutions. So the study of the stability is not complete. Moreover, the method of Lyapunov function is adopted in this paper to study the global stability. It should be pointed out that the global stability can be improved in certain circumstance.

On the other hand, we are interested in the existence of nonconstant positive stationary solutions of (1.4), which is called stationary pattern. In the past decades, much work has been devoted to the investigation of the existence of stationary pattern in chemical and biological dynamics theoretically as well as numerically [10,11,14,17,20–22,26,28]. The existence of nonconstant positive stationary solution is usually mathematically challenging, which is focused on in this paper. By the linear stability analysis, it is shown that as (1.4) has two constant positive stationary solutions, at least one of them is unstable. So, nonconstant positive stationary solutions may be expected. By the theory of Leray-Schauder degree, we show that there may exist nonconstant positive stationary solutions in this case. It should be pointed out that a priori estimate of positive stationary solutions plays an important role in the analysis. In particular, good a priori estimate is deduced, which has no restriction on the coefficients in the reaction terms.

The rest of this paper is organized as follows. In Section 2, we study the properties of constant positive stationary solutions of system (1.4). In Section 3, nonexistence and existence of nonconstant positive stationary solutions are shown.

2. Properties of constant positive stationary solution

In this section, we show some properties of the constant positive stationary solution of (1.4), including the nonexistence, existence, multiplicity and stability.

First, it is clear that the stationary problem of (1.4) is given by

$$\begin{cases} -d_1\Delta u = u\left(\lambda - u - \dfrac{\beta v}{u+mv}\right), & x \in \Omega, \\ -d_2\Delta v = v\left(1 - \dfrac{v}{u+k}\right), & x \in \Omega, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega. \end{cases} \tag{2.1}$$

Then (u, v) is a constant positive stationary solution of (1.4) if and only if

$$\beta = \frac{(\lambda - u)[(1+m)u + mk]}{u+k} = h(u) \quad \text{for} \quad u \in (0, \lambda). \tag{2.2}$$

Then it can be verified that

$$h'(u) = \frac{-(1+m)u^2 - 2k(1+m)u + k(\lambda - mk)}{(u+k)^2}. \tag{2.3}$$

By some routine analysis, we can deduce the following result:

Lemma 2.1. *For the function $h(u)$ defined by (2.2), we have that*

(i) if $\lambda \le mk$, $h(u)$ is strictly decreasing for $u \in (0, \lambda)$;

(ii) if $\lambda > mk$, then there exits a unique positive number $\hat{u} \in (0, \lambda)$ such that

$$\begin{cases} h'(u) > 0, & u \in (0, \hat{u}), \\ h'(u) < 0, & u \in (\hat{u}, \lambda). \end{cases}$$

Here, $\hat{u}$ is given by

$$\hat{u} = -k + \sqrt{\frac{k(k+\lambda)}{1+m}}.$$

Theorem 2.1. *(i) If $\lambda \le mk$, then (2.1) has constant positive solutions if and only if $0 < \beta < m\lambda$. Moreover, it is unique.*

(ii) If $\lambda > mk$, then (2.1) has constant positive solutions if and only if $0 < \beta \le h(\hat{u})$. Moreover, as $0 < \beta \le m\lambda$ or $\beta = h(\hat{u})$, the constant positive solution is unique; as $m\lambda < \beta < h(\hat{u})$, there exist two constant positive solutions. Here, h and $\hat{u}$ are given by Lemma 2.1.

It should be pointed out that when (2.1) has a unique constant positive solution, we always denote it by (u_*, v_*); when there exist two constant positive solutions, we always denote them by (u_{1*}, v_{1*}) and (u_{2*}, v_{2*}) with $u_{1*} < u_{2*}$.

Next, we consider the stability of the constant positive solution of (2.1). To do so, let $0 = \mu_0 < \mu_1 < \mu_2 < \cdots$ be the eigenvalues of $-\Delta$ on Ω with homogeneous Neumann boundary condition. Then for

$$X = \left\{(u, v) \in \left[C^1(\bar{\Omega})\right]^2 : \partial_\nu u|_{\partial\Omega} = \partial_\nu v|_{\partial\Omega} = 0\right\}, \tag{2.4}$$

we can decompose X as

$$X = \bigoplus_{i=0}^{\infty} X_i,$$

where X_i is the eigenspace corresponding to μ_i. Let

$$\mathbf{u} = (u, v)^T, \ D = \operatorname{diag}(d_1, d_2) \tag{2.5}$$

and

$$F(\mathbf{u}) = \left(u\left(\lambda - u - \frac{\beta v}{u + mv}\right), v\left(1 - \frac{v}{u+k}\right)\right)^T. \tag{2.6}$$

Then (2.1) can be rewritten by

$$D\Delta \mathbf{u} + F(\mathbf{u}) = \mathbf{0}. \tag{2.7}$$

Let $\bar{\mathbf{u}}_* = (\bar{u}_*, \bar{v}_*)$ be a constant positive solution of (2.1). Then the linearized problem of (2.7) at $\bar{\mathbf{u}}_*$ is given by

$$D\Delta \mathbf{u} + F_{\mathbf{u}}(\bar{\mathbf{u}}_*)\mathbf{u} = \mu \mathbf{u},$$

where

$$F_{\mathbf{u}}(\bar{\mathbf{u}}_*) = \begin{pmatrix} \lambda - 2\bar{u}_* - \frac{m\beta \bar{v}_*^2}{(\bar{u}_*+m\bar{v}_*)^2} & -\frac{\beta \bar{u}_*^2}{(\bar{u}_*+m\bar{v}_*)^2} \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} A(\bar{\mathbf{u}}_*) & B(\bar{\mathbf{u}}_*) \\ 1 & -1 \end{pmatrix}. \tag{2.8}$$

Since X_i is invariant under the operator $D\Delta + F_{\mathbf{u}}(\bar{\mathbf{u}}_*)$, we obtain that μ is an eigenvalue of the linearized problem on X_i if and only if it is an eigenvalue of the following matrix

$$-\mu_i D + F_{\mathbf{u}}(\bar{\mathbf{u}}_*) = \begin{pmatrix} -d_1\mu_i + A(\bar{\mathbf{u}}_*) & B(\bar{\mathbf{u}}_*) \\ 1 & -1 - d_2\mu_i \end{pmatrix}.$$

So, the characteristic equation is given by

$$\mu^2 - T_i(\bar{\mathbf{u}}_*)\mu + D_i(\bar{\mathbf{u}}_*) = 0,$$

where

$$T_i(\bar{\mathbf{u}}_*) = -(d_1 + d_2)\mu_i - 1 + A(\bar{\mathbf{u}}_*),$$

$$D_i(\bar{\mathbf{u}}_*) = d_1 d_2 {\mu_i}^2 + [d_1 - d_2 A(\bar{\mathbf{u}}_*)]\, \mu_i - (A(\bar{\mathbf{u}}_*) + B(\bar{\mathbf{u}}_*)).$$

It is clear that $B(\bar{\mathbf{u}}_*) < 0$.

To show the stability of $\bar{\mathbf{u}}_*$, we need to consider the sign of $A(\bar{\mathbf{u}}_*)$ and $-(A(\bar{\mathbf{u}}_*)+B(\bar{\mathbf{u}}_*))$.

Lemma 2.2. *Assume that $\bar{\mathbf{u}}_* = (\bar{u}_*, \bar{v}_*)$ is a constant positive solution of* (2.1). *Then*

(i) $(A(\bar{\mathbf{u}}_*) + B(\bar{\mathbf{u}}_*))h'(\bar{u}_*) > 0$, *where h is the function given by Lemma 2.1;*

(ii) *if $\lambda \le mk$, we have that* $A(\bar{\mathbf{u}}_*) < 0$;

(iii) *if $\lambda > mk$, we have that*

$$\begin{cases} A(\bar{\mathbf{u}}_*) < 0, & \text{if} \quad u_* > \tilde{u}, \\ A(\bar{\mathbf{u}}_*) > 0, & \text{if} \quad u_* < \tilde{u}, \end{cases}$$

where $\tilde{u}$ is the positive number given by

$$\tilde{u} = \frac{\lambda - mk}{m+2}. \tag{2.9}$$

Proof. By some computations, we obtain that

$$\begin{aligned}
-(A(\bar{\mathbf{u}}_*)+B(\bar{\mathbf{u}}_*)) &= \frac{\beta\bar{u}_*^2}{(\bar{u}_*+m\bar{v}_*)^2} - \left[\lambda - 2\bar{u}_* - \frac{m\beta\bar{v}_*^2}{(\bar{u}_*+m\bar{v}_*)^2}\right] \\
&= \frac{h(\bar{u}_*)\bar{u}_*^2}{(\bar{u}_*+m\bar{v}_*)^2} - \left[\lambda - 2\bar{u}_* - \frac{mh(\bar{u}_*)\bar{v}_*^2}{(\bar{u}_*+m\bar{v}_*)^2}\right] \\
&= \frac{-\bar{u}_*(\bar{u}_*+k)h'(\bar{u}_*)}{(1+m)\bar{u}_*+mk},
\end{aligned}$$

which yields the conclusion of (i).

For the sign of $A(\bar{\mathbf{u}}_*)$, we can deduce that

$$\begin{aligned}
A(\bar{\mathbf{u}}_*) &= \lambda - 2\bar{u}_* - \frac{m\beta\bar{v}_*^2}{(\bar{u}_*+m\bar{v}_*)^2} \\
&= \frac{\bar{u}_*}{(1+m)\bar{u}_*+mk}\left[(\lambda-mk)-(m+2)\bar{u}_*\right].
\end{aligned}$$

Thus, the conclusions of (ii) and (iii) are obtained. The proof of the lemma is complete. □

It should be remarked that by some computations, we can show that

$$h'(\tilde{u}) = \frac{\lambda-mk}{(\tilde{u}+k)^2(m+2)^2}\left[-km(m+2)-(1+m)(\lambda-mk)\right] < 0.$$

Thus, it follows that

$$\hat{u} < \tilde{u} < \frac{\lambda-mk}{1+m},$$

where $(\lambda-mk)/(1+m)$ is the unique constant positive solution of (2.2) for $\beta = m\lambda$.

Theorem 2.2. *(i) Assume that $\lambda \le mk$, $0<\beta<m\lambda$ or $\lambda > mk$, $0<\beta\le m\lambda$, then the unique constant positive solution (u_*, v_*) of* (2.1) *is asymptotically stable.*

(ii) Assume that $\lambda > mk$ and $m\lambda < \beta \le h(\tilde{u})$ with $\tilde{u}$ given by (2.9). *Then (u_{1*}, v_{1*}) is unstable and (u_{2*}, v_{2*}) is stable.*

(iii) Assume that $\lambda > mk$ and $h(\tilde{u}) < \beta < h(\hat{u})$. Then (u_{1*}, v_{1*}) is unstable. If $d_1\mu_j < A(u_{2*})$ for some positive integer j and d_2 is large enough, then (u_{2*}, v_{2*}) is unstable; if $d_1\mu_1 > A(u_{2*})$, then (u_{2*}, v_{2*}) is stable for $A(u_{2*}) > 1$ and unstable for $A(u_{2*}) < 1$.

Proof. First, if $\lambda \le mk$ and $0<\beta<m\lambda$, Lemma 2.2 asserts that $A(u_*, v_*) < 0$. If $\lambda > mk$ and $0 < \beta \le m\lambda$, we have that

$$\frac{\lambda-mk}{1+m} \le u_* < \lambda.$$

So, it follows that $u_* > \tilde{u}$, which implies that $A(u_*, v_*) < 0$. Thus, $T_i(\mathbf{u}_*) < 0$ and $D_i(\mathbf{u}_*) > 0$ for any $i \ge 0$. The conclusion of (1) is proved.

Second, if $\lambda > mk$ and $m\lambda < \beta \le h(\tilde{u})$, (2.1) has two constant positive solutions (u_{i*}, v_{i*}) $(i=1,2)$ with $u_{1*} < u_{2*}$. Moreover,

$$0 < u_{1*} < \hat{u} < \tilde{u} \le u_{2*} < \frac{\lambda-mk}{1+m}.$$

Then $h'(u_{1*}) > 0$. By virtue of Lemma 2.2, we have that

$$D_0(u_{1*}, v_{1*}) = -(A(u_{1*}, v_{1*}) + B(u_{1*}, v_{1*})) < 0,$$

which implies that (u_{1*}, v_{1*}) is unstable. On the other hand, as $u_{2*} > \tilde{u}$, one sees that $A(u_{2*}, v_{2*}) < 0$, which yields that (u_{2*}, v_{2*}) is stable.

Finally, if $\lambda > mk$ and $h(\tilde{u}) < \beta < h(\hat{u})$, one can also see that (2.1) has two constant positive solutions (u_{i*}, v_{i*}) $(i = 1, 2)$ with $u_{1*} < \hat{u} < u_{2*} < \tilde{u}$. Thus, (u_{1*}, v_{1*}) is unstable. Moreover, we have that $A(u_{2*}, v_{2*}) > 0$ and $D_0(u_{2*}, v_{2*}) > 0$. As

$$D_j(u_{2*}, v_{2*}) = d_2\mu_j(d_1\mu_j - A(u_{2*}, v_{2*})) + d_1\mu_j - D_0(u_{2*}, v_{2*}),$$

it is easy to see the conclusion of (3). Thus, the proof of the theorem is complete. □

Finally, we show the global stability of the constant positive solution of (2.1).

Theorem 2.3. *Assume that $\lambda < mk$ and $0 < \beta < m\lambda$. Then the unique constant positive solution $\boldsymbol{u}_* = (u_*, v_*)$ of (2.1) is globally asymptotically stable.*

Proof. Let $(u(x,t), v(x,t))$ be the solution of (2.1). Set

$$f(u, v) = \lambda - u - \frac{\beta v}{u + mv}, \quad g(u, v) = 1 - \frac{v}{u + k}$$

and define

$$W(u, v) = \int \frac{u - u_*}{u} du + \alpha \int \frac{v - v_*}{v} dv, \quad E(t) = \int_\Omega W(u(x,t), v(x,t)) dx,$$

where α is a positive constant to be decided later.

Direct computations give that

$$\begin{aligned}
\frac{dE(t)}{dt} &= \int_\Omega \{W_u(u(x,t), v(x,t))u_t + W_v(u(x,t), v(x,t))v_t\} \, dx \\
&= \int_\Omega \left\{ \frac{u - u_*}{u} [d_1 \Delta u + u f(u, v)] + \alpha \frac{v - v*}{v} [d_2 \Delta v + v g(u, v)] \right\} dx \\
&= -\int_\Omega \left(d_1 \frac{u_*}{u^2} |\nabla u|^2 + \alpha d_2 \frac{v_*}{v^2} |\nabla v|^2 \right) dx \\
&\quad + \int_\Omega [(u - u_*) f(u, v) + \alpha(v - v_*) g(u, v)] \, dx \\
&= I_1(t) + I_2(t).
\end{aligned}$$

It is obvious that

$$I_1(t) = -\int_\Omega \left(d_1 \frac{u_*}{u^2} |\nabla u|^2 + \alpha d_2 \frac{v_*}{v^2} |\nabla v|^2 \right) dx \le 0.$$

Furthermore, we can deduce that

$$\begin{aligned}
I_2(t) = \int_\Omega \Bigg\{ &(u - u_*)^2 \left[-1 + \frac{\beta v_*}{(u + mv)(u_* + mv_*)} \right] \\
&+ \left[\frac{\alpha}{u + k} - \frac{\beta u_*}{(u + mv)(u_* + mv_*)} \right] (u - u_*)(v - v_*) - \frac{\alpha}{u + k} (v - v_*)^2 \Bigg\} dx.
\end{aligned}$$

If there exist positive constants α and T such that for all $x \in \bar{\Omega}$ and $t \geq T$, any solution $(u(x,t), v(x,t))$ of (2.1) satisfies

$$-1 + \frac{\beta v_*}{(u+mv)(u_*+mv_*)} < 0$$

and

$$\left[\frac{\alpha}{u+k} - \frac{\beta u_*}{(u+mv)(u_*+mv_*)}\right]^2 + 4\frac{\alpha}{u+k}\left[-1 + \frac{\beta v_*}{(u+mv)(u_*+mv_*)}\right] < 0, \tag{2.10}$$

then

$$\frac{dE(t)}{dt} = I_1(t) + I_2(t) \leq 0$$

for $t \geq T$, which implies that $\mathbf{u}_* = (u_*, v_*)$ is globally asymptotically stable.

On the one hand, a standard comparison argument asserts that for

$$0 < \varepsilon < \min\left\{k, \frac{k(m\lambda - \beta)}{mk+\beta}\right\},$$

there exists $T > 0$ such that for all $x \in \overline{\Omega}$ and $t > T$,

$$u(x,t) > \lambda - \frac{\beta}{m} - \varepsilon, \quad v(x,t) > k - \varepsilon.$$

Thus, for $t > T$, we have that

$$\begin{aligned}
(u+mv)(u_*+mv_*) &> \left[\left(\lambda - \frac{\beta}{m} - \varepsilon\right) + m(k-\varepsilon)\right] mv_* \\
&> \left(\lambda - \frac{\beta}{m} - \varepsilon + \frac{k-\varepsilon}{k}\lambda\right) mv_* \\
&> \left(m\lambda - \beta - m\varepsilon + \frac{k-\varepsilon}{k}\beta\right) v_* \\
&= \beta v_* + \left[m\lambda - \beta - \left(m + \frac{\beta}{k}\right)\varepsilon\right] v_* > \beta v_*.
\end{aligned}$$

On the other hand, since

$$\begin{aligned}
\psi(\alpha) &= \left[\frac{\alpha}{u+k} - \frac{\beta u_*}{(u+mv)(u_*+mv_*)}\right]^2 + 4\frac{\alpha}{u+k}\left[-1 + \frac{\beta v_*}{(u+mv)(u_*+mv_*)}\right] \\
&= \left[\frac{\alpha}{u+k} + \frac{\beta u_*}{(u+mv)(u_*+mv_*)}\right]^2 + 4\left[\frac{\beta k}{(u+mv)(u_*+mv_*)} - 1\right]\frac{\alpha}{u+k},
\end{aligned}$$

one sees that $\psi(\alpha) < 0$ is equivalent to

$$\frac{\alpha}{u+k} + \frac{\beta u_*}{(u+mv)(u_*+mv_*)} < 2\sqrt{\alpha}\sqrt{\frac{1}{u+k}\left[1 - \frac{\beta k}{(u+mv)(u_*+mv_*)}\right]}. \tag{2.11}$$

While it can be verified that

$$\triangle = 4\frac{1}{u+k}\left[1 - \frac{\beta k}{(u+mv)(u_*+mv_*)}\right] - 4\frac{1}{u+k}\frac{\beta u_*}{(u+mv)(u_*+mv_*)}$$

$$= 4\frac{1}{u+k}\left[1 - \frac{\beta v_*}{(u+mv)(u_*+mv_*)}\right] > 0$$

for $t > T$ and $x \in \bar{\Omega}$, which implies that there exists $\alpha > 0$ such that $\psi(\alpha) < 0$ for $t > T$ and $x \in \bar{\Omega}$. Thus, the proof is complete. □

Remark 2.1. Due to Theorem 2.1, one sees that (1.4) actually has a unique constant positive stationary solution for $0 < \beta < m\lambda$. In [29], Zhao and Bie also considered system (1.4). By the method of upper-lower solutions, they deduced that the unique constant positive stationary solution is globally stable when $m < 1$. By Theorem 2.3, we show that the global stability is true for $\lambda < mk$. Thus, as $\lambda/k > 1$, the unique constant positive stationary solution is globally stable for $m < 1$ and $m > \lambda/k$, which implies that we supplement the result of global stability; as $\lambda/k < 1$, the unique constant positive stationary solution is globally stable for any $m > \beta/\lambda$, which implies that we improve the existing result of global stability.

3. Properties of nonconstant positive stationary solutions

In this section, we investigate the nonexistence and existence of nonconstant positive solutions of (2.1). Before that, we need to establish a priori estimate of positive solutions of (2.1).

First, in order to obtain a priori estimates, we list two useful lemmas due to Lou and Ni [17] and Lin et al. [16], respectively.

Lemma 3.1. (Maximum Principle [17]) *Suppose that* $g \in C(\bar{\Omega} \times \mathbb{R})$.

(i) Assume that $w(x) \in C^2(\Omega) \cap C^1(\bar{\Omega})$ *satisfies*

$$\Delta w(x) + g(x, w(x)) \geq 0 \text{ in } \Omega, \quad \partial_\nu w|_{\partial\Omega} \leq 0.$$

If $w(x_0) = \max_{\bar{\Omega}} w(x)$, *then* $g(x_0, w(x_0)) \geq 0$.

(ii) Assume that $w(x) \in C^2(\Omega) \cap C^1(\bar{\Omega})$ *satisfies*

$$\Delta w(x) + g(x, w(x)) \leq 0 \text{ in } \Omega, \quad \partial_\nu w|_{\partial\Omega} \geq 0.$$

If $w(x_0) = \min_{\bar{\Omega}} w(x)$, *then* $g(x_0, w(x_0)) \leq 0$.

Lemma 3.2. (Harnack Inequality [16]) *Assume that* $c(x) \in C(\bar{\Omega})$, $w(x) \in C^2(\Omega) \cap C^1(\bar{\Omega})$ *is a positive solution to*

$$\Delta w(x) + c(x)w(x) = 0 \text{ in } \Omega, \quad \partial_\nu w|_{\partial\Omega} = 0.$$

Then there exists a positive constant $C^* = C^*(\Omega, \|c(x)\|_\infty)$ *such that*

$$\max_{\bar{\Omega}} w(x) \leq C^* \min_{\bar{\Omega}} w(x).$$

For notational convenience, we denote the constants m, k, λ, β collected by Λ in the following.

Theorem 3.1. *Assume that m, k, λ, β are fixed constants. For any given positive constant κ, there exist two positive constants $\underline{C}(\Lambda,\kappa,\Omega)$ and $\overline{C}(\Lambda,\kappa,\Omega)$ such that if $d_1 \geq \kappa$, any positive solution (u, v) of (2.1) satisfies*

$$\underline{C} < u(x), v(x) < \overline{C}, \quad x \in \bar{\Omega}.$$

Proof. A direct application of Lemma 3.2 to the first equation of (2.1) yields that $u(x) < \lambda$. Let $v(x_0) = \max_{\bar{\Omega}} v(x)$, then the maximum principle shows that

$$v(x_0) \leq u(x_0) + k \leq k + \max_{\bar{\Omega}} u(x) < \lambda + k. \tag{3.1}$$

Then the desired positive upper bound $\overline{C}$ is found.

Let $v(x_1) = \min_{\bar{\Omega}} v(x)$, the maximum principle yields that

$$\frac{v(x_1)}{u(x_1) + k} \geq 1.$$

Therefore, it follows that

$$v(x) \geq v(x_1) \geq u(x_1) + k > k. \tag{3.2}$$

So, if the conclusion is false, there exists a sequence $\{d_{1n}\}_{n\geq 1}$ with $d_{1n} \geq \kappa$ and a positive solution (u_n, v_n) of (2.1) corresponding to $d_1 = d_{1n}$ satisfies

$$\min_{\bar{\Omega}} u_n \to 0 \text{ as } n \to \infty.$$

It is clear that u_n satisfies

$$\Delta u_n + \frac{u_n}{d_{1n}}(\lambda - u_n - \frac{\beta v_n}{u_n + m v_n}) = 0. \tag{3.3}$$

Since

$$\|\frac{1}{d_{1n}}\left(\lambda - u_n - \frac{\beta v_n}{u_n + m v_n}\right)\| \leq \frac{1}{\kappa}(2\lambda + \frac{\beta}{m}),$$

the Harnack inequality asserts that there exists a positive constant C independent of n such that $\max_{\bar{\Omega}} u_n(x) \leq C \min_{\bar{\Omega}} u_n(x)$. Thus, we have that $u_n(x) \to 0$ uniformly as $n \to \infty$. As $k < v_n < k + \max_{\bar{\Omega}} u_n(x)$, it follows that $v_n(x) \to k$ uniformly as $n \to \infty$.

Integrating the equation of (3.3) over Ω, we obtain that

$$\int_{\Omega} u_n \left(\lambda - u_n - \frac{\beta v_n}{u_n + m v_n}\right) dx = 0. \tag{3.4}$$

So, one sees that

$$\lambda - u_n - \frac{\beta v_n}{u_n + m v_n} \to \lambda - \frac{\beta}{m} \text{ as } n \to \infty.$$

If $\lambda \neq \frac{\beta}{m}$, then a contradiction to (3.4) is derived.

If $\lambda = \frac{\beta}{m}$ and $\beta \neq m^2 k$, by some rearrangements, we see that

$$0 = \int_{\Omega} u_n \left(\lambda - u_n - \frac{\beta v_n}{u_n + m v_n}\right) dx = \int_{\Omega} {u_n}^2 \left(\frac{\beta}{m}\frac{1}{u_n + m v_n} - 1\right) dx.$$

It is clear that

$$\frac{\beta}{m}\frac{1}{u_n+mv_n}-1\to\frac{\beta}{m^2k}-1 \text{ as } n\to\infty.$$

Thus, we also deduce a contradiction.

If $\lambda=\frac{\beta}{m}$ and $\beta=m^2k$, some rearrangements yield that

$$\begin{aligned}0=\int_\Omega u_n\left(\lambda-u_n-\frac{\beta v_n}{u_n+mv_n}\right)dx&=\int_\Omega {u_n}^2\left(\frac{\beta}{m}\frac{1}{u_n+mv_n}-\frac{\beta}{m^2k}\right)dx\\&=-\frac{\beta}{m^2k}\int_\Omega u_n^2\frac{u_n+m(v_n-k)}{u_n+mv_n}dx.\end{aligned}$$

Since $v_n>k$, one sees that for sufficiently large n,

$$\int_\Omega u_n\left(\lambda-u_n-\frac{\beta v_n}{u_n+mv_n}\right)dx=-\frac{\beta}{m^2k}\int_\Omega u_n^2\frac{u_n+m(v_n-k)}{u_n+mv_n}dx<0,$$

which is a contradiction to (3.4). Thus, the proof of the theorem is complete. □

Next, we use the energy method to show the nonexistence of nonconstant positive solutions of (2.1).

Theorem 3.2. *Assume that m, k, λ, β and d_2 are fixed constants with $d_2\ge\kappa_0>1/\mu_1$. Then there exists a positive constant κ_1 depending on m, k, λ, β and κ_0 such that* (2.1) *has no nonconstant positive solutions for $d_1\ge\kappa_1$.*

Proof. Assume that (u,v) is a positive solution of (2.1). Denote

$$\bar u=\frac{1}{|\Omega|}\int_\Omega udx,\qquad \bar v=\frac{1}{|\Omega|}\int_\Omega vdx.$$

Multiplying $(u-\bar u)$ to the first equation of (2.1) and integrating on Ω, we obtain that

$$\begin{aligned}d_1\int_\Omega|\nabla u|^2dx&=\int_\Omega\left\{(u-\bar u)\left[u(\lambda-u)-\frac{\beta uv}{u+mv}-\bar u(\lambda-\bar u)+\frac{\beta\bar u\bar v}{\bar u+m\bar v}\right]\right\}dx\\&=\int_\Omega\left\{\left[\lambda-(u+\bar u)-\frac{m\beta v\bar v}{(u+mv)(\bar u+m\bar v)}\right](u-\bar u)^2\right.\\&\qquad\left.-\frac{\beta u\bar u}{(u+mv)(\bar u+m\bar v)}(u-\bar u)(v-\bar v)\right\}dx\\&\le\int_\Omega\left\{\lambda(u-\bar u)^2-\frac{\beta u\bar u}{(u+mv)(\bar u+m\bar v)}(u-\bar u)(v-\bar v)\right\}dx.\end{aligned}$$

Similarly, we have

$$\begin{aligned}d_2\int_\Omega|\nabla v|^2dx&=\int_\Omega\left\{(v-\bar v)^2\left(1-\frac{v+\bar v}{\bar u+k}\right)+\frac{v^2}{(u+k)(\bar u+k)}(u-\bar u)(v-\bar v)\right\}dx\\&\le\int_\Omega\left\{(v-\bar v)^2+\frac{v^2}{(u+k)(\bar u+k)}(u-\bar u)(v-\bar v)\right\}dx.\end{aligned}$$

Since

$$\left\|\frac{v^2}{(u+k)(\bar u+k)}-\frac{\beta u\bar u}{(u+mv)(\bar u+m\bar v)}\right\|<\beta+\frac{(k+\lambda)^2}{k^2},$$

it follows that there exists a positive constant M such that

$$\int_\Omega \left(d_1|\nabla u|^2 + d_2|\nabla v|^2\right) dx \le \int_\Omega \left\{\lambda(u-\bar{u})^2 + 2M|u-\bar{u}||v-\bar{v}| + (v-\bar{v})^2\right\} dx.$$

Applying ε-Young Inequality, we have that

$$\int_\Omega \left(d_1|\nabla u|^2 + d_2|\nabla v|^2\right) dx \le \int_\Omega \left\{\left(\lambda + \frac{M}{\varepsilon}\right)(u-\bar{u})^2 + (1+M\varepsilon)(v-\bar{v})^2\right\} dx,$$

where ε is any positive number. Using the Poincaré Inequality, we have

$$\int_\Omega \left(d_1\mu_1|u-\bar{u}|^2 + d_2\mu_1|v-\bar{v}|^2\right) dx \le \int_\Omega \left\{\left(\lambda + \frac{M}{\varepsilon}\right)(u-\bar{u})^2 + (1+M\varepsilon)(v-\bar{v})^2\right\} dx.$$

As $d_2 \ge \kappa_0 > 1/\mu_1$, we choose

$$0 < \varepsilon_0 < \frac{\kappa_0\mu_1 - 1}{M}, \quad \kappa_1 = \frac{1}{\mu_1}(\lambda + \frac{M}{\varepsilon_0}).$$

Then for $d_1 \ge \kappa_1$, we can conclude that u and v are constants. Thus, the proof of the theorem is complete. □

Finally, we show the existence of nonconstant positive solutions of (2.1) by using the Leray-Schauder degree theory. For $\mathbf{u}, D$ and $F(\mathbf{u})$ defined by (2.5) and(2.6), it is easy to see that $\mathbf{u}$ is a solution of (2.1) if and only if it satisfies

$$G(d_1, d_2; \mathbf{u}) := \mathbf{u} - (I-\Delta)^{-1}\left\{D^{-1}F(\mathbf{u}) + \mathbf{u}\right\} = \mathbf{0} \text{ on } X. \tag{3.5}$$

Here, $(I-\Delta)^{-1}$ is the inverse operator of $I-\Delta$ in Ω with the homogeneous Neumann boundary condition, X is given by (2.4).

If $\mathbf{u}_*$ is a constant positive solution of (3.5), some direct computations deduce that

$$G_{\mathbf{u}}(d_1, d_2; \mathbf{u}_*) = I - (I-\Delta)^{-1}(D^{-1}F_{\mathbf{u}}(\mathbf{u}_*) + I).$$

If $G_{\mathbf{u}}(d_1, d_2; \mathbf{u}_*)$ is invertible, then the index of G at $\mathbf{u}_*$ is defined by

$$\text{index}(G, \mathbf{u}_*) = (-1)^\sigma,$$

where σ is the multiplicity of negative eigenvalues of $G_{\mathbf{u}}(\mathbf{u}_*)$. As in [20], we define

$$H(d_1, d_2; \mu; \mathbf{u}_*) = \det(\mu D - F_{\mathbf{u}}(\mathbf{u}_*)). \tag{3.6}$$

Then it can be shown that

$$H(d_1, d_2; \mu) = d_1 d_2 \mu^2 + (d_1 - A d_2)\,\mu - (A+B),$$

where A and B are given by (2.8). Then

$$\lim_{d_2\to\infty} \frac{H(\mu)}{d_2} = \mu(d_1\mu - A).$$

By virtue of Lemma 2.2, we can obtain the following result:

Lemma 3.3. *Assume that* $\lambda > mk$ *and* $m\lambda < \beta < h(\hat{u})$, (2.1) *has two constant positive solutions* $\mathbf{u}_{i*} = (u_{i*}, v_{i*})$ $(i = 1, 2)$. *Then as* d_2 *is large enough,* (3.6) *with* $\mathbf{u}_*$ *has two real roots* ${\mu_j}^-$ *and* ${\mu_j}^+$. *For* $\mathbf{u}_* = \mathbf{u}_{1*}$, *we have that*

$$ {\mu_j}^- < 0 < {\mu_j}^+, \quad \lim_{d_2 \to \infty} {\mu_j}^+ = \frac{A(\mathbf{u}_{1*})}{d_1}. $$

For $\mathbf{u}_* = \mathbf{u}_{2*}$, *we have that if* $m\lambda < \beta < h(\tilde{u})$, *then* ${\mu_j}^- < {\mu_j}^+ < 0$; *if* $h(\tilde{u}) < \beta < h(\hat{u})$, *then* $0 < {\mu_j}^- < {\mu_j}^+$ *with the property*

$$ \lim_{d_2 \to \infty} {\mu_j}^- = 0, \quad \lim_{d_2 \to \infty} {\mu_j}^+ = \frac{A(\mathbf{u}_{2*})}{d_1}. $$

Now we can give the following existence results:

Theorem 3.3. *Assume that* $\lambda > mk$ *and* $h(\tilde{u}) < \beta < h(\hat{u})$. *If* $\frac{A(\mathbf{u}_{1*})}{d_1} \in (\mu_p, \mu_{p+1})$ *and* $\frac{A(\mathbf{u}_{2*})}{d_1} \in (\mu_q, \mu_{q+1})$ *for some positive integers* p *and* q, *and* $\sum_{i=1}^{p} m(\mu_i) + \sum_{i=1}^{q} m(\mu_i)$ *is odd, where* $m(\mu_i)$ *is the multiplicity of* μ_i, *then there exists a positive constant* ${d_2}^*$ *such that system* (2.1) *has at least one nonconstant positive solution for all* $d_2 > {d_2}^*$.

Proof. First, due to Theorem 2.1 and Lemma 3.3, one sees that system (2.1) has two constant positive solutions $\mathbf{u}_{i*} = (u_{i*}, v_{i*})(i = 1, 2)$ with

$$ u_{1*} < u_{2*}, \quad A(\mathbf{u}_{1*}) > 0, \quad A(\mathbf{u}_{2*}) > 0. $$

As $\frac{A(\mathbf{u}_{1*})}{d_1} \in (\mu_p, \mu_{p+1})$ and $\frac{A(\mathbf{u}_{2*})}{d_1} \in (\mu_q, \mu_{q+1})$, then there exists some positive number d_* such that for $d_2 > d_*$, we have that

$$ H(d_1, d_2; \mu; \mathbf{u}_{1*}) < 0 \quad \text{for } \mu = \mu_0, \mu_1, \cdots, \mu_p $$

and

$$ H(d_1, d_2; \mu; \mathbf{u}_{2*}) < 0 \quad \text{for } \mu = \mu_1, \mu_2, \cdots, \mu_q. $$

If the conclusion is not true, then there exists some $\hat{d}_2$ such that system (2.1) with $d_2 = \hat{d}_2$ has no nonconstant positive solutions. Then (2.1) with $d_2 = \hat{d}_2$ has two constant positive solutions $\mathbf{u}_{i*}(i = 1, 2)$. Moreover, Theorem 3.2 asserts that there exists some large positive number $\hat{d}_1 > d_*$ such that (2.1) with $d_1 = \hat{d}_1$ and $d_2 \geq \hat{d}_1$ has no nonconstant positive solutions and exactly two constant positive solutions $\mathbf{u}_{i*}(i = 1, 2)$. Choosing $\hat{d}_1$ large enough such that $\frac{A(\mathbf{u}_{i*})}{\hat{d}_1} < \mu_1$ for $i = 1, 2$, it follows that

$$ H(\hat{d}_1, \hat{d}_1; \mu; \mathbf{u}_{1*}) < 0 \quad \text{for } \mu = \mu_0 $$

and

$$ H(\hat{d}_1, \hat{d}_1; \mu; \mathbf{u}_{2*}) > 0 \quad \text{for any } \mu_i. $$

For $t \in [0, 1]$, we define

$$ D(t) = \begin{pmatrix} td_1 + (1-t)\widehat{d_1} & 0 \\ 0 & t\widehat{d_2} + (1-t)\widehat{d_1} \end{pmatrix}, \quad t \in [0, 1], $$

and

$$\Phi(\mathbf{u}, t) = \mathbf{u} - (I - \Delta)^{-1} \left\{ D^{-1}(t) F(\mathbf{u}) + \mathbf{u} \right\} = \mathbf{0} \quad on \quad X. \tag{3.7}$$

Then, $\mathbf{u}$ is a positive solution of (2.1) if and only if it is a positive solution of equation (3.7) with $t = 1$. Moreover, Theorem 3.1 deduces that there exist positive constants $\underline{C}$ and $\overline{C}$ such that any nonnegative solution of (3.7) satisfies $\underline{C} < u(x), v(x) < \overline{C}$ for all $t \in [0, 1]$. Thus, for M defined by

$$M = \left\{ \mathbf{u} = (u, v) \in X : \frac{1}{2}\underline{C} < u(x), v(x) < 2\overline{C} \right\},$$

we have that $\Phi(\mathbf{u}, t) \neq 0$ for all $\mathbf{u} \in \partial M$ and $t \in [0, 1]$. Then the homotopy invariance of Leray-Schauder degree yields that

$$\deg(\Phi(\cdot, 1), M, 0) = \deg(\Phi(\cdot, 0), M, 0).$$

On the one hand,

$$\deg(\Phi(\cdot, 0), M, 0) = \text{index}(\Phi(\cdot; 0), \mathbf{u}_{1*}) + \text{index}(\Phi(\cdot; 0), \mathbf{u}_{2*}) = (-1)^1 + (-1)^0 = 0.$$

On the other hand,

$$\begin{aligned} \deg(\Phi(\cdot, 1), M, 0) &= \text{index}(\Phi(\cdot; 1), \mathbf{u}_{1*}) + \text{index}(\Phi(\cdot; 1), \mathbf{u}_{2*}) \\ &= (-1)^{1 + \sum_{i=1}^{p} m(\mu_i)} + (-1)^{\sum_{i=1}^{q} m(\mu_i)} = -2 \text{ or } 2. \end{aligned}$$

Thus, we derive a contradiction. So, the proof of the theorem is complete. □

Similarly, we can deduce the following conclusion.

Theorem 3.4. *Assume that* $\lambda > mk$, $m\lambda < \beta < h(\tilde{u})$. *If* $\frac{A(\mathbf{u}_{1*})}{d_1} \in (\mu_p, \mu_{p+1})$ *for some positive integer* p, *and* $\sum_{i=1}^{p} m(\mu_i)$ *is odd, then there exists a positive constant* ${d_2}^*$ *such that system* (2.1) *has at least one nonconstant positive solution for all* $d_2 > {d_2}^*$.

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