Journal of Nonlinear Modeling and Analysis http://jnma.ca; http://jnma.ijournal.cn
Volume 3, Number 3, September 2021, 335–348 DOI:10.12150/jnma.2021.335

Optimal Value and Post Optimal Solution in a Transportation Problem

Tolulope Latunde[1,†], Joseph Oluwaseun Richard[1], Opeyemi Odunayo Esan[1] and Damilola Deborah Dare[1]

Abstract In this work, we analyse the transportation problem of a real-life situation by obtaining the optimal feasible solutions, thus carrying out the sensitivity analysis of the problem. The work utilises the data obtained from the Asejire and Ikeja plants of Coca-Cola company, aiming to aid decision-making regarding the best possible options to satisfy customers at the barest minimum cost of transportation. Rerunning the optimization of a problem is an expensive scheme for gathering and obtaining enough data required for a problem. Thus, to minimize the transportation cost, the sensitivity analysis of parameters is a good tool to determine the behaviour of some input parameters where the values of these parameters are varied arbitrarily such that optimal results are verified. Maple 18 Software is used to solve the problem and the result obtained is compared with the values evaluated from northwest corner method, least cost method and Vogel's approximation method. The study critically shows how a little change in a unit or more of any model parameter affects the expected results.

Keywords Optimal solution, Sensitivity analysis, Transportation.

MSC(2010) 90B06, 90C31.

1. Introduction

Transportation problem can generally be defined as the problem of how to structuring production and mobilize goods produced from the factories. This can either be regarded as source or origin or park as the case may be, at different destinations and larger to their customers all across wherever they might be. This is the reason why it is regarded as transportation problem. The real-life problem that can yet make light of but very stressful is the problem called transportation problem for companies and organization especially for the procurement department of the manufacturing and transport companies.

There is no permanent solution ideal enough to be a remedy to life problems. However, mathematics fully assured us that problems also have solutions. Thus, more methods will be developed, and applications built and different tools will be created to control most life problems.

The transportation problem began to form a shape as a problem in 1871 by a French economist and mathematician called Gaspard Monge, and the transportation problem was first studied in 1920 as a problem according to Sarbjit (2012) [20].

†the corresponding author.
Email address: tolulope.latunde@fuoye.edu.ng (T. Latunde)
[1]Department of Mathematics, Federal University Oye-Ekiti, Nigeria

One of the key problems that companies or organizations are facing is transportation problems. Ralf et al. [16] (1998) concluded that one way of optimization is better planning. They used methods that have been developed in the theory of optimization to maximize the result of resources and existing technology. The work of Ralf et al. [16] shows that discrete mathematics applies to the theory of optimization, inputting powerful algorithm and putting modern computations into planning practice. The transportation problem applies to industries, companies, communication network, genetics, transportation schedule and allotment. The transportation models or problems are primarily concerned with how a product can be best transported from different factories or plants (origins) to some warehouses (destinations). The goal of every transportation problem is to reach the requirements of the destination, with which the capacity constraints at the minimum possible cost of production operates. It is necessary to understand that movement of goods from any source to any destination will require that cost is being minimized, if income (profit) must be maximized. Let us take the below as an example. Nine distinct factories such as X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8 and X_9 have to meet the request of also nine warehouses, say Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8 and Y_9. The goods available at each factory is specified x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 and x_9, and the goods requested at each warehouse y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8 and y_9. The cost of moving goods from the factories to warehouses can be represented on tables of saying m_{ij} where the subscripts (1-9) indicate the cell, given the cost of moving from the factories (origin) i to the warehouse destination) j. Therefore, the cost of moving goods from the factory x_4 to warehouse y_7 is m_{47}. The transportation problem is the problem of how to plan production and transport goods produced from the plants (source/origin/park) at different locations and larger to their customers all across wherever they might be. This is the reason for the name-transportation problem because many of its applications are involved in the objective of transporting goods with the best possible outcome. The problem that can yet be unnoticed but very frustrating is the logistical problem called transportation problem for organizations, especially for manufacturing and transport companies. The linear programming method is a useful tool for dealing with such a problem as a transportation problem. Each source can supply a fixed number of units of products, which is usually called capacity or availability. Each destination has a fixed demand, which is usually known as a requirement. The nature of the and its application in solving problems involving several products from sources to several destinations, and this type of problem is frequently and generally called “The Transportation Problem”.

2. Literature review

In the research work “Optimization Techniques for Transportation Problem of Three Variables”, Rekha [18] applied four methods named northwest corner method, least cost method, Vogel and MODI method. In the process of considering the optimization techniques of transportation for three variables, the steps to each method and the steps to determine the optimal solution were explained, and the comparison between the MODI method and every other method was made. The work aims at getting the shortest, best and cheapest route to satisfy the demand from any destination. In the paper “A Comparative study of transportation problem under probabilistic and fuzzy uncertainties”, Chaudhuri and Kajal [3] worked on the comparative study of transportation problem by using probabilistic and fuzzy uncertainties, and they accomplished an enhanced ability to develop a cost-effective solution to transportation problems. Olson believes that the lack of an efficient algorithm for the model solution is a great limitation [14], and he worked upon this by comparing his results and schemes with the existing algorithms for accuracy and time requirements. He concluded that the dual simplex has an edge over other methods compared in computational times when it comes to a large proportion of variables in the solutions. Shraddha gave several methods of solving the transportation problem and obtain the objective of evaluating the similarities and differences of these methods [21]. In his work "Solving Transportation Problem by Various Methods and their Comparison", Shraddha solved the transportation problem of the Millennium Herbal Company by using different methods and comparing the results yielded. Meanwhile, in the work entitled "A Case Study of The Optimization of The Transportation Cost for Raipur Steel and Thermal Power Plant", Dharmendra and Saurabh used the TORA, LINGO and What's Best Solver to determine the transportation cost, and the data used were collected manually analysed and compared with [6].

In the work "Load flow optimization and optimal power flow", Das dealt with the cost coefficient of the objective function thoroughly by seeing into the multi-objective problems of transportation [5]. The fuzzy programming technique was applied to solve the problem that was converted from constraints to a deterministic solvable problem.

Bit et al. introduced an additive fuzzy programming model for the multi-objective transportation problem [2]. The method aggregates the membership functions of the objectives to construct the relevant decision function. Weights and priorities for non-equivalent objectives are also incorporated in the method. Their model gave a non-dominated solution that is nearer to the best-compromised solution. Rao, in the fourth chapter "Engineering Optimization: Theory and Practice", explained the concept of linear programming, the revised simplex method, duality in linear programming under which he worked on duality theorems and dual simplex method. The fourth chapter discussed more on the sensitivity analysis or post optimality of optimisation problems such as Transportation Problem. However, Rao concluded that there are two major methods of obtaining solutions for practical aspects of optimization, which is firstly the sensitivity equation using Kuhn-Tucker conditions and secondly sensitivity equation using the concept of feasible direction.

The determination of paradoxical pairs in a linear transport problem deals with the efficiency of an algorithm for obtaining the solution to LP problems [7]. In the work, Ekezie et al. concluded that the paradox exists. They used the northwest corner method to obtain the optimal feasible solution to the problem with the aid of TORA Statistical Software Package. It was an algorithm worth studying for deriving the solution procedure for finding all the paradoxical pairs.

Asase also worked on some transportation problems and application to real-life situations [1]. He utilised GUINNESS GHANA LIMITED in Kumasi as a case study where it was explicitly explained, Vogel's approximation method, northwest corner method, least cost method as well as for test for optimality, and he dealt with Stepping Stone Method and Modified Distribution Method (MODI). Asase created a table for the problem formulation and solved it by using Management Scientist 5.0.

Dantzig used the simplex method to determine the solution of transportation as the primal simplex transportation [4]. He proposed that the initial basic feasible solution for the transportation problem can be determined through Column Minima Method, Row minima, Matrix minima, Vogel's approximation method and northwest corner rule. For the optimal solution, he used Modified Distribution (MODI).

Lee et al. used the goal programming to solve the multi-objective problem of transportation [11]. They noticed that most models that were earlier developed to find the solution to transportation neglect the multiple conflicting objectives involving such a problem. For the purpose of optimizing multiple conflicting goals putting the decision of the existing environment into consideration, Lee et al. considered it was necessary to study the goal programming. Bureaucratic decision structures and unique organizational values of the firm and environmental constraints are the prioritized structures of the objectives.

By studying the application of transportation linear programming algorithms to cost reduction in Nigeria soft drinks industry by Salami [19], it was deduced from the analysis of the data collected from the Distribution Department of 7-up Bottling Company PLC using three distinct methods yields the same result as the transportation cost. He hereby concludes and recommends to the 7-up Bottling Company PLC that any of the three methods can be adopted.

In their book "Mathematical Methods on Optimization in Transportation Systems", Matti and Jarklo [12] shared their work into two distinct parts: (i) Public Transport Models; (ii) General Transport Models. In the first part, they carefully dealt with the prevention of delay in railway traffic by optimization and simulation. Then, the heuristic for scheduling buses and drivers for a former urban public transport computing with bus-driver dependencies. They worked on the functionality of the simulation program Simu+++ and Dispo+++, which was developed at the Institute of Transport, Railway Construction and Operation in the University of Hanover, Germany.

A new method for optimal solutions to the transportation of problems is possible. However, it may not be too complicated to understand and carry out in the time of the study personally. Muhammad and Farzana gave a feature of their work entitled "A new method for optimal solutions of transportation problem in LPP" [13] that requires very simple arithmetical and logical calculation, and even so a layman can pick and understand. They cited several examples and developed a programming code that was efficient enough to tackle such problems if encountered.

Susanta applied a new approach called Palsu's Favorable Cost Method, and gave some numerical examples with illustration in his work "An Optimal Solution for Transportation Problem: Direct Approach" [22], this method is useful to directly solve the transportation problem without finding its initial basic feasible solution, he compared the result with northwest corner method, row minimum Method, least cost method and Vogel's approximation method. Then, Palsu's favorable cost method had been concluded.

In this work, we utilised northwest corner method, least cost method and Vogel's approximation method, sharing similarity with solving directly by using Maple Software. There are some other commercial and academic computational software that is efficient and very useful such as TORA, MATLAB, GUROBI package, Linear Interactive and Discrete Optimization (LINDO), General Interactive Optimizer (GINO) and so on.

One edge of sensitivity analysis of parameters over every other model for obtaining the results of increased parameters is the ability to re-optimize as many times as possible and a new model can be drawn and confirmed to be very effective for the work at hand and recommendable for others. In their work "Sensitivity Analysis in Solid Transportation Problems [15]", Pandian and Kavitha proposed a new bound technique for cost sensitivity ranging from solid transportation problems. According to Latunde et al. (2016), Latunde and Bamigbola (2018), Latunde et al. (2019) and Latunde et al. (2020), some other optimization methods have also been applied to real-life situations in engineering, asset management, financial sector and transportation, where sensitivity analysis of model parameters in the system designs was carried out, and control policies such as [9], [8] and [10] were suggested.

3. Mathematical formulation

The purpose of this work is to optimize the total cost of transportation from Asejire Factory and Ikeja Factory to study different distributors in Ibadan, Oyo State. The cost of transportation of a full truck of 1,530 crates of beverages are expressed in Nigeria Naira (million), and the quantity of goods demand and the supply is determined by the number of crates. The information above is for 12 months from April 2017 to March 2018. Suppose a company has$x$ warehouses and the number of retailers to be y, we can only ship one product from x to y.

We can build a mathematical model for the following transportation problem. For example, see Table 1 below, where X_i implies the supply to the warehouses, and Y_j is the demand by the retailer outlets.

Table 1. Sample

	X_1	X_2	X_3	X_4	Supply
Y_1	1	2	4	4	6
Y_2	4	3	2	0	8
Y_3	0	2	2	1	10
Demand	4	6	8	6	24

Let companies produce goods at different places (factories) say, 'm' factories from i=1,2,3,...,m. Meanwhile, it supplies to different distributors or warehouses, we have this to be S_i, i=1,2,3,...,n. The demand from the factory reaches all requested places (e.g. wholesalers). The demand from the last wholesaler is the jth place, and we call this D_j.

The problem of the company is to get goods from factory i and supply to the wholesaler j, the cost is c_{ij} and this transportation cost is *linear*. By formulation, if we transport a_{ij} numbers of goods from factory i to wholesaler j, the cost is $c_{ij}a_{ij}$.

The problem is to figure out the minimum cost of transporting those goods. The condition that must be satisfied here is that we must meet the demand at each of the wholesalers' request and supply cannot exceed. Therefore, linearly, the cost of this program is:

$$min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} a_{ij} \tag{3.1}$$

The number of goods transported from the factory i is

$$\sum_{j=i}^{n} a_{ij}. \tag{3.2}$$

The goods cannot be more than the request to be supplied to the wholesaler. Therefore, we have it that

$$\sum_{j=i}^{n} \leq S_i \forall i = 1, 2, ..., m. \tag{3.3}$$

Similarly, the constraints to make sure demand is met at all wholesalers point is.

$$\sum_{i=i}^{n} \geq D_j \forall j = 1, 2, ..., n \tag{3.4}$$

There would be excessive demand, if the sum of all supply is not more than the demand as such, and the request from the wholesalers will be much after supplies have been made to avoid this. Therefore, we have that

$$\sum_{i=i}^{n} D_j \leq \sum_{i=i}^{m} S_i. \tag{3.5}$$

If this is not holding, the demand cannot be met. Hence, there must be enough possibly excessive supply to be sure that demand is met.

It is also fair to assume that the quantities demand is exactly equal to the quantities supplied.

$$\sum_{i=i}^{n} D_j = \sum_{i=i}^{m} S_i, \tag{3.6}$$

When this happens, it means that the plan for transportation cost is perfect and the supply meets the wholesalers' need at every point and disposed of all goods that left the factory. Therefore, at the cost c_{ij}, m supplies for i=1,2,3,...,m, S_i and n demands D_j for j=1,2,3,...,n.

The major work is to find a transportation schedule denoted by x_{ij} to get a solution to

$$min \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}, \tag{3.7}$$

subject to

$$\sum_{j=i}^{n} = S_i \forall i = 1, 2, ..., m, \tag{3.8}$$

and also to

$$\sum_{i=i}^{n} = D_j \forall j = 1, 2, ..., n. \tag{3.9}$$

3.1. Data collection and analysis

The Coca-Cola Company has been known as the world's largest beverage company. Coca-Cola owns and markets more than 500 nonalcoholic beverage brands.

These are categorised into:

1. Sparkling soft drinks;
2. Water and enhanced water;
3. Sports drinks (juice and dairy);
4. Plant-based beverages (tea and coffee);
5. Energy drinks.

Like every other beverage company, the Coca-Cola PLC is available in over 200 countries and transports goods based on the network of company-owned or -controlled bottling and distributions operations, independent bottling partners, distributors, wholesalers and retailers.

Table 2. Table representation of the problem (in thousand units)

	FID	Akin	Oniyele	MGR	Adhex	FDR	Vero	BnB	Mimz	Nuhi	Ile-Iwe	Supply
Asejire	206	182	242	277	196	212	200	276	192	150	303	1320
Ikeja	180	206	235	261	177	197	212	255	200	198	295	1210
Demand	**200**	**200**	**250**	**280**	**200**	**220**	**220**	**270**	**180**	**210**	**300**	**2530**

The following depots are presented represented in Table 2:
FID – Fidelity Depot
Akin – Akin Depot
Oniyele – Oniyele Depot
MGR – Madam Margaret Depot
Adhex – Adhex Depot
FDR – Fadare and Sons Depot
Vero – Veronica Depot
BnB – Bolu and Bola Depot
Mimz – Mimz Depot
Nuhi – Nuhi Depot
Ile-Iwe – Ile-Iwe Depot

3.2. Problem formulation

Let Y_1= factory at Asejire and Y_2= the factory at Ikeja.

X_{ij} = the units transported in crates from factories i to warehouses j respectively

i= $1, 2, 3...m, n.$ and j= $1, 2, 3...,m, n.$

Therefore, x_{11} represents the units shipped from Asejire Plant to FID Warehouse, x_{12} implies to Akin up to x_{1m} which is from Asejire to Nuhi and lastly x_{1n} which is to Ile-Iwe.

Same as above, x_{21} represents the units shipped from Ikeja Plant to FID Warehouse, x_{22} ps from Ikeja to Akin up to x_{2m} which is from Ikeja to Nuhi and lastly from Ikeja is x_{2n} which is to Ile-Iwe.

Based on the information in Table 2, the 12-month transportation cost can be considered as:

Min Z = $206x_{11} + 182x_{12} + 242x_{13} + 277x_{14} + 196x_{15} + 212x_{16} + 200x_{17} + 276x_{18} + 192x_{19} + 150x_{1m} + 303x_{1n} + 180x_{21} + 206x_{22} + 235x_{23} + 261x_{24} + 177x_{25} + 197x_{26} + 212x_{27} + 255x_{28} + 200x_{29} + 198x_{2m} + 295x_{2n}$

Subject to:

The available supply constraint is given by

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{1m} + x_{1n} \leq 1320$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{25} + x_{27} + x_{28} + x_{29} + x_{2m} + x_{2n} \leq 1210$$

Like the demand constraint is as computed below:

$$x_{11} + x_{21} \leq 200$$

$$x_{12} + x_{22} \leq 200$$

$$x_{13} + x_{23} \leq 250$$

$$x_{14} + x_{24} \leq 280$$

$$x_{15} + x_{25} \leq 200$$

$$x_{16} + x_{26} \leq 220$$

$$x_{17} + x_{27} \leq 220$$

$$x_{18} + x_{28} \leq 270$$

$$x_{19} + x_{29} \leq 180$$

$$x_{1m} + x_{2m} \leq 210$$

$$x_{1n} + x_{2n} \leq 300$$

$$x_{1i}, x_{2i} > 0$$

$\forall$ i = 1,2. j = 1,2...m,n.

3.3. Application of northwest corner method

Table 3. Table representation of northwest corner method (in thousand units)

	FID	Akin	Oniyele	MGR	Adhex	FDR	Vero	BnB	Mimz	Nuhi	Ile-Iwe	Supply
Asejire	200	200	250	280	200	190						1320
Ikeja						30	220	270	180	210	300	1210
Demand	**200**	**200**	**250**	**280**	**200**	**220**	**220**	**270**	**180**	**210**	**300**	**2530**

Therefore, the transportation cost for northwest corner method is $(206 \times 200) + (182 \times 200) + (243 \times 250) + (277 \times 280) + (196 \times 200) + (212 \times 190) + (197 \times 30) + (212 \times 220) + (255 \times 270) + (200 \times 180) + (215 \times 300)$ = 41,200 + 36,400 + 60,500 + 72,560 + 39,200 + 40,280 + 5,910 + 46,640 + 68,850 + 36,000 + 64,500 = 517,040 units.

3.4. Application of least cost method

Table 4. Table representation of least cost method (in thousand units)

	FID	Akin	Oniyele	MGR	Adhex	FDR	Vero	BnB	Mimz	Nuhi	Ile-Iwe	Supply
Asejire		200		280			220	230	180	210		1320
Ikeja	200		250		200	220		40			300	1210
Demand	**200**	**200**	**250**	**280**	**200**	**220**	**220**	**270**	**180**	**210**	**300**	**2530**

Therefore, the transportation cost for least cost method is $(206 \times 200) + (182 \times 200) + (243 \times 250) + (277 \times 280) + (196 \times 200) + (212 \times 190) + (197 \times 30) + (212 \times 220) + (255 \times 270) + (200 \times 180) + (215 \times 300) = 36{,}400 + 77{,}560 + 44{,}000 + 63{,}480 + 34{,}560 + 31{,}500 + 36{,}000 + 58{,}750 + 35{,}400 + 43{,}340 + 10{,}200 + 64{,}500 = 535{,}690$ units.

3.5. Application of Vogel's approximation method

Table 5. Table representation of least cost method (in thousand units)

	FID	Akin	Oniyele	MGR	Adhex	FDR	Vero	BnB	Mimz	Nuhi	Ile-Iwe	Supply
Asejire	200	200					220	10	180	210	300	1320
Ikeja			250	280	200	220		260				1210
Demand	**200**	**200**	**250**	**280**	**200**	**220**	**220**	**270**	**180**	**210**	**300**	**2530**

Therefore, the transportation cost for Vogel's approximation method is $(206 \times 200) + (182 \times 200) + (200 \times 220) + (192 \times 180) + (210 \times 150) + (303 \times 300) + (235 \times 250) + (261 \times 280) + (200 \times 172) + (220 \times 197) + (260 \times 255) = 41{,}200 + 34{,}400 + 44{,}000 + 2{,}760 + 34{,}560 + 90{,}900 + 36{,}000 + 58{,}750 + 73{,}080 + 34{,}340 + 43{,}430 + 66{,}300 = 525{,}690$ units.

Also, by solving the problem on MAPLE software, we obtained the optimized value of the minimum transportation cost 546,919 units.

Table 6 represents the results obtained by each method used in solving the transportation problem.

Table 6. Table representation of method of solution and cost value (in thousand)

Method	Cost Value
Northwest Corner Method	517,040
Least Cost Method	535,690
Vogel's Approximation Method	525,690
Maple	546,919

In Table 6 above, we have that the northwest corner method produces the best transportation cost for this problem. Since the northwest corner method yields a transportation cost of 517,040, least cost method produces 535,690, and Vogel's approximation method gives 525,690. The computer solution shows the minimum total transportation cost computed by MAPLE is 546,919.

The reduced cost of value percentage =

$$\frac{InitialCost - OptimizedCost}{InitialCost} \times 100 \tag{3.10}$$

For the initial transportation cost of delivering goods with respect to demand, we have

$(206 \times 200) + (182 \times 200) + (242 \times 250) + (277 \times 280) + (196 \times 200) + (212 \times 220) + (212 \times 220) + (276 \times 270) + (200 \times 180) + (198 \times 210) + (303 \times 300) = 41{,}200 + 36{,}400 + 60{,}500 + 77{,}560 + 39{,}200 + 46{,}640 + 46{,}640 + 74{,}520 + 36{,}000 + 41{,}580 + 90{,}900 = 591{,}140$ units.

For NWC, the reduced cost is

$$\frac{591,140 - 517,040}{591,140} \times 100\% = 12.54\%$$

For LCM, the reduced cost is

$$\frac{591,140 - 535,690}{591,140} \times 100\% = 9.38\%$$

For VAM, the reduced cost is

$$\frac{591,140 - 525,690}{591,140} \times 100\% = 11.07\%$$

For MAPLE, the reduced cost is

$$\frac{591,140 - 546,919}{591,140} \times 100\% = 7.48\%$$

The problem of Coca-Cola PLC was solved with three distinct methods named northwest corner method, least cost method and Vogel's approximation method, and then compared with the result computed by the linear programming module called MAPLE.

It is obtained that the Northwest corner method produces the optimum transportation cost with 517,040 and corresponding reduced cost of the problem as 12.5%.

3.6. Result discussion on goods transportation

In this work, NWC is north west corner, VAM is Vogel's approximation method and LWC represents least cost method. The problem of Coca-Cola PLC was solved with three distinct methods named northwest corner method, least cost method and Vogel's approximation method, and then compared with the result computed by the linear programming module called Maple 18 software. Table 6 shows that the northwest corner method produces the optimum transportation cost which is 517,040.

4. Sensitivity analysis

The solution to the Coca-Cola PLC problem was obtained by using three distinct methods named northwest corner method, least cost method and Vogel's approximation method, and then compared with the result computed by the linear programming module called MAPLE.

Thus, northwest corner method produces the optimum transportation cost which is 517,040.

Based on the analysis here, we increase the number of cases demanded by each warehouse from both plants (Asejire and Ikeja), we do this by adding 50 cases each. We run every addition by the MAPLE software to determine the outcome and the optimized cost that will be generated, it was studied that some warehouses had more cost value than the others.

Consider Table 7 below showing the results of the sensitivity analysis of the post optimal solutions to the transportation problems.

Table 7. Result of sensitivity analysis from the Maple Programmer

S/N	Warehouse	Number of Old Cases	Number of New Cases	Optimized Result	RCV %
1	$\boldsymbol{X_{11}}$	206	256	506,420	14.77
2	$\boldsymbol{X_{12}}$	182	232	556,920	6.27
3	$\boldsymbol{X_{13}}$	242	292	559,420	5.85
4	$\boldsymbol{X_{14}}$	277	327	560,920	5.60
5	$\boldsymbol{X_{15}}$	196	246	557,920	6.10
6	$\boldsymbol{X_{16}}$	212	262	557,920	6.10
7	$\boldsymbol{X_{17}}$	200	250	560,420	5.68
8	$\boldsymbol{X_{18}}$	276	326	555,920	6.44
9	$\boldsymbol{X_{19}}$	192	242	557,420	6.19
10	$\boldsymbol{X_{1m}}$	150	200	561,920	5.43
11	$\boldsymbol{X_{1n}}$	303	353	597,420	-0.54

12	$\boldsymbol{X_{21}}$	180	230	546,920	7.96
13	$\boldsymbol{X_{22}}$	206	256	546,920	7.96
14	$\boldsymbol{X_{23}}$	235	285	546,920	7.96
15	$\boldsymbol{X_{24}}$	261	301	556,920	6.27
16	$\boldsymbol{X_{25}}$	177	257	546,920	7.96
17	$\boldsymbol{X_{26}}$	197	247	546,920	7.96
18	$\boldsymbol{X_{27}}$	212	262	546,920	7.96
19	$\boldsymbol{X_{28}}$	255	305	546,920	7.96
20	$\boldsymbol{X_{29}}$	200	250	546,920	7.96
21	$\boldsymbol{X_{2m}}$	198	248	546,920	7.96
22	$\boldsymbol{X_{2n}}$	295	345	546,920	7.96

In Table 7, above X_{11} is the number of cases demanded by FID from Asejire Plant, X_{12} is the number of cases demanded by Akin also from the Asejire plant, it continues as represented in Tables 3-5 and continues on the row until X_{1m}, which is the number of cases demanded by Nuhi and lastly X_{1n} which is Ile-Iwe.

X_{21} is the number of cases demanded by FID from Ikeja Plant, X_{22} is the number of cases demanded by Akin also from the Ikeja plant, it continues as represented in Table 3-5 and continues on the row until X_{1m} which is the number of cases demanded from Ikeja by Nuhi and lastly X_{1n} which is Ile-Iwe.

The result of sensitivity analysis shows that more cases of drinks can and should be supplied to X_{11}, which is the FID warehouse from Asejire as it has the largest optimum reduced cost value almost twice of the others. The implication of this is that by priority of supply, X_{11} will get more cases and will still minimize cost.

From Table 7, it is also obtained that X_{11} has the highest percentage of reduced cost value and X_{1n} has the least, this can be interpreted that the higher the Optimized Result, the less the reduced cost value and vice-versa are.

5. Conclusion

Since the transportation problem is one of the major problems in the optimization field, operation research and even life problems to live companies. The transportation problem was formulated as linear programming and solved with Maple software. The computational results provided the minimal total transportation cost and the values that will optimize the cost of supplying, the number of cases to supply and where to supply more cases.

The study shows that the best method that will save the highest percentage of transportation cost for this problem in the northwest corner method with 12.54%. For goods transportation, the computational results provided the minimal total transportation cost and the values that will optimize the cost of supplying, the number of cases to supply and where to supply more cases.

Also, more cases are advised to be supplied to FID from the Asejire Plant for the optimum reduced value of transportation cost. Supplying 50 extra cases to FID more than other warehouses will reduce by 14.77%.

As for future directions, the sensitivity analysis of this work may be carried out by using a different and more sophisticated approach. Similarly, the formulated transportation problem may be optimised from the perspective of multi-objective optimisation problems.

References

[1] A. Asase, *The transportation problem, Case Study: Guinness Ghana Limited* (Master's thesis), Faculty of Physical Science and Technology Kumasi, Ghana, 2011.

[2] A. K. Bit, M. P. Biswal and S. S. Alam, *Fuzzy programming to multi-objective solid transportation problem*, Fuzzy Sets and Systems, 1993, 57, 183–194.

[3] A. Chaudhuri and D. Kajal, *A Comparative study of transportation problem under probabilistic and fuzzy uncertainties*, Journal of Mathematics and Computer Science, 2013, 256–267.

[4] G. B. Dantzig, *Linear Programming and Extensions*, Princeton University Press, Princeton, 1963.

[5] J. C. Das, *Load flow optimization and optimal power flow*, CRC Press, Boca Raton, 2017.

[6] Y. Dharmendra and K. Saurabh, *A case study of the optimization of the transportation cost for Raipur steel and thermal power plant*, International Journal for Research in Applied Science and Engineering Technology, 2017, 5(9).
DOI: 10.22214/ijraset.2017.9113

[7] D. D. Ekezie, J. O. Chukwudi and J. Opara, *The determination of paradoxical pairs in a linear transport problem*, International Journal of Mathematics and Statistics Studies, 2013, 1(3), 9–19.

[8] T. Latunde and O. M. Bamigbola, *Parameter estimation and sensitivity analysis of an optimal control model for capital asset management*, Advances in Fuzzy Systems, 2018, 1–11.
DOI: 10.1155/2018/4756520

[9] T. Latunde, O. M. Bamigbola and Y. O. Aderinto, *Sensitivity of parameters in an optimal control model of the electric power generating system*, Ilorin Journal of Computer Science and Information Technology, 2016, 1(1), 54–70.

[10] T. Latunde, J. O. Richard, O. O. Esan and D. D. Dare, *Sensitivity of parameters in the approach of linear programming to a transportation problem*, Journal of the Nigerian Society of Physical Science, 2019, 1(4), 116–121.

[11] S. M. Lee and L. J. Moore, *Optimization transportation problems with multiple objectives*, A I I R Transactions, 1973, 5(4), 333–338.
DOI: 10.1080/05695557308974920

[12] P. Matti and N. Jarklo, *Mathematical methods on optimization in transportation systems*, 2001.
DOI: 10.1007/978-1-4757-3357-0

[13] H. Muhammad and S. R. Farzana, *A new method for optimal solutions of transportation problem in LPP*, Journal of Mathematics Research, 2018, 5. DOI: 10.5539/jmr.v10n5p60

[14] D. L. Olson, *Comparison for four goal programming algorithm*, Journal of Operational Research Society, 1984, 35(4), 347–354.

[15] P. Pandian and K. Kavitha, *Sensitivity analysis in solid transportation problems*, Applied Mathematical Sciences, 2012, 6(136), 6787–6796.

[16] B. Ralf, G. Martins and L. Andreas, *Optimization of transportation System*, Konrad-Zuse-Zentrum für Informationstechnik, 1998, ZIB-Report 98-09.

[17] S. S. Rao, *Engineering Optimization Theory and Practice*, John Wiley & Sons, Hoboken, 2009.

[18] V. J. Rekha, *Optimization techniques for transportation problem of three variables*, International Organization of Scientific Research (IOSR) Journal of Mathematics, 2013, 9(1), 46–50.

[19] A. O. Salami, *Application of transportation linear programming algorithms to cost reduction in Nigeria soft drinks industry*, International Journal of Economics and Management Engineering, 2014, 8(2), 416–422.

[20] S. Sarbjit, *Note on transportation problem with new method for resolution of degeneracy*, Universal Journal of Industrial and Business Management, 2012, 3(1), 26–36.

[21] M. Shraddha, *Solving transportation problem by various method and their comparison*, International Journal of Mathematics Trends and Technology, 2017, 2231–5373.

[22] K. M. Susanta, *An optimal solution for transportation problem: Direct approach*, 2019. DOI: 10.13140/RG.2.2.19271.34720