Journal of Nonlinear Modeling and Analysis
Volume 3, Number 3, September 2021, 365–376

http://jnma.ca; http://jnma.ijournal.cn
DOI:10.12150/jnma.2021.365

Lump Solutions, Lump-soliton Interaction Phenomena and Breather-soliton Solutions to a (3+1)-dimensional Boiti-Leon-Manna-Pempinelli-like Equation*

Huiqin Xu[1] and Yinshan Yun[1,†]

Abstract The (3+1)-dimensional Boiti-Leon-Manna-Pempinelli-like equation (BLMP-like equation) is introduced by the generalized bilinear operators D_p associated with $p = 3$. The lump solutions, lump-soliton interaction phenomena and breather-soliton solutions are discussed to the (3+1)-dimensional BLMP-like equation based on the generalized bilinear method with symbolic computation system Mathematica. In order to observe the behavior of those solutions, we fix the value of z, then give the 3D-graphs of some solutions at different times. We find a lump solution moved in oblique direction; a lump-soliton interaction phenomenon is appeared and disappeared along with the time. We also see a kink-breather soliton moved in oblique direction.

Keywords Interaction phenomena, Lump solution, (3+1)-dimensional Boiti-Leon-Manna-Pempinelli-like equation.

MSC(2010) 34G20, 35C08, 39A14.

1. Introduction

Many physical and mechanical phenomena are nonlinear. In this way, solving nonlinear differential equations has become a hot issue in the field of applied mathematics. So far, many researchers have proposed and developed various techniques for solving nonlinear differential equations, such as Lie symmetry [3, 35], Hirota bilinear method [4, 12, 20, 21, 25, 30], homotopy perturbation method [10, 11, 33, 34], homotopy analysis method [13], the Adomian decomposition method [1, 27] (ADM), auxiliary equation methods [8, 32] and so on.

Among those techniques, the Hirota bilinear method was proposed by Japan's famous mathematician and physicist Ryogo Hirota [12] for solving nonlinear differential equations. If a nonlinear differential equation has the bilinear form, one can find its many physical characteristics by its many kinds of exact solutions. Based on the Hirota bilinear operator, in recent years, researchers found lump solutions of some differential equations to explain many physical phenomena in various subjects, including plasma physics, shallow water waves and optical media [9, 26, 28].

†the corresponding author.
Email: yunyinshan@126.com (Y. Yun)
[1]College of Sciences, Inner Mongolia University of Technology, Hohhot, Inner Mongolia 010051, China
*The authors were supported by Natural Science Foundation of Inner Mongolia Autonomous Region (No. 2020LH01003).

Based on Hirota bilinear operators, Ma [17–19] proposed the generalized bilinear differential operators as follows:

$$\begin{aligned}
&D_{p,x}^{m}D_{p,t}^{n}f\cdot f\\
&=\left(\frac{\partial}{\partial x}+\alpha_p\frac{\partial}{\partial x'}\right)^{m}\left(\frac{\partial}{\partial t}+\alpha_p\frac{\partial}{\partial t'}\right)^{n}f(x,t)f(x',t')\Big|_{\substack{x'=x\\ t'=t}}\\
&=\sum_{i=0}^{m}\sum_{j=0}^{n}\binom{m}{i}\binom{n}{j}\alpha_p^i\alpha_p^j\frac{\partial^{m-i}}{\partial x^{m-i}}\frac{\partial^{i}}{\partial x'^{i}}\frac{\partial^{n-j}}{\partial t^{n-j}}\frac{\partial^{j}}{\partial t'^{j}}f(x,t)f(x',t')\Big|_{\substack{x'=x\\ t'=t}}\\
&=\sum_{i=0}^{m}\sum_{j=0}^{n}\binom{m}{i}\binom{n}{j}\alpha_p^i\alpha_p^j\frac{\partial^{m+n-i-j}f(x,t)}{\partial x^{m-i}t^{n-j}}\frac{\partial^{i+j}f(x,t)}{\partial x^{i}t^{j}},m,n\geq 0,
\end{aligned}\tag{1.1}$$

where the α_p^s satisfies:

$$\alpha_p^s=(-1)^{r_p(s)},\quad s=r_p(s)\ \mathrm{mod}\ p\,,$$

and

$$\alpha_p^i\alpha_p^j\neq\alpha_p^{i+j},\quad i,j\geq 0\,,$$

when the prime number $p\geq 2$. For different prime numbers p, Hirota bilinear equations have been generalized to generate diverse nonlinear differential equations possessing potential applications. In recent years, based on the generalized bilinear differential operators, the lump solutions, rational solution and interaction solutions are discussed. For example, new periodic wave, cross-kink wave and the interaction phenomenon for the Jimbo-Miwa-like equation [36], lump solutions with higher-order rational dispersion to the second KPI equation [23], lump and interaction solutions to linear PDEs in 2+1 dimensions [22], determining lump solutions for a combined soliton equation in (2+1)-dimensions [29], lump solutions to a combined equation involving three types of nonlinear terms relations [24], lump solutions to a (2+1)-dimensional fifth-order KdV-like equation [2], lump solutions to dimensionally reduced Kadomtsev-Petviashvili-like equations [31], interaction solutions to the (3+1)-dimensional Kadomtsev-Petviashvili-Boussinesq-like equation [15], rational solutions to two Sawada Kotera-like equations [6], rational solutions to a Hirota-Satsuma-like equation [14], rational solutions to an extended Kadomtsev-Petviashvili-like equation [16], rational solutions to a Kdv-like Equation [37] and rational solutions to a generalized (2+1)-dimensional Shallow-Water-Wave-like equation [7].

In this paper, based on the above techniques, we would like to introduce a BLMP-like equation by the generalized bilinear operators. Then, we will discuss its lump solutions, lump-soliton interaction phenomena and breather-soliton solutions.

2. The (3+1)-dimensional BLMP-like equation

The (3+1)-dimensional BLMP equation [5] is as follows:

$$u_{yt}+u_{zt}+u_{xxxy}+u_{xxxz}-3u_x(u_{xy}+uxz)-3u_{xx}(u_y+u_z)=0.$$

Its bilinear equation is as follows:

$$B_{BLMP}(f) := (D_y D_t + D_z D_t + D_y D_x^3 + D_z D_x^3) f \cdot f.$$

Based on the generalized bilinear operator (1.1), when prime number $p = 3$, the generalized bilinear equation is

$$\begin{aligned} B_{BLMP-like}(f) &:= (D_{3,y} D_{3,t} + D_{3,z} D_{3,t} + D_{3,y} D_{3,x}^3 + D_{3,z} D_{3,x}^3) f \cdot f \\ &= -2 f_t f_y + 2 f f_{yt} - 2 f_t f_z + 2 f f_{zt} + 6 f_{xy} f_{xx} \\ &\quad - 6 f_x f_{xxy} + 6 f_{xz} f_{xx} - 6 f_x f_{xxz} = 0. \end{aligned} \tag{2.1}$$

Through the bilinear transformation $u = -2(lnf)_x$, the BLMP-like equation is generated as follows:

$$\begin{aligned} &-4u_{zt} - 4u_{yt} + 6uu_z u_x + 6uu_y u_x + 3u^2 u_{xz} + 3u^2 u_{xy} \\ &\qquad + 6u_z u_{xx} + 6u_y u_{xx} - 6uu_{xxz} - 6uu_{xxy} = 0. \end{aligned} \tag{2.2}$$

3. Lump solutions of the BLMP-like equation

In order to obtain lump solutions, we suppose

$$\begin{aligned} f &= g^2 + h^2 + a_{11} \\ g &= a_1 x + a_2 y + a_3 z + a_4 t + a_5 \\ h &= a_6 x + a_7 y + a_8 z + a_9 t + a_{10}, \end{aligned} \tag{3.1}$$

where $a_i (i = 1, 2, ...11)$ are real constants. With the help of mathematics, by substituting (3.1) into Equation (2.1), we obtain a set of algebraic equations about $a_i (i = 1, 2, ...11)$. Solving the set of algebraic equations, we obtain as the following three-kind lump solutions of the BLMP-like equation.

Case (1):

$$\begin{aligned} &a_1 = a_1, a_2 = a_2, a_3 = -a_2, a_4 = a_4, a_5 = a_5, a_6 = a_6, \\ &a_7 = -a_8, a_8 = a_8, a_9 = a_9, a_{10} = a_{10}, a_{11} = a_{11}, \end{aligned} \tag{3.2}$$

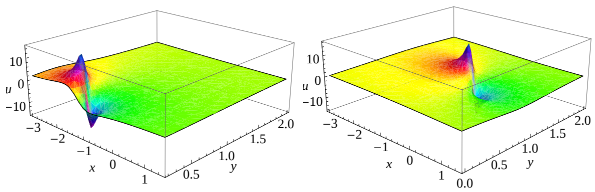
where $a_i (i = 1, 2, 4, 5, 6, 8, 9, 10, 11)$ are arbitrary constants, the corresponding solution to BLMP-like equation (2.2) is as follows:

$$u(x, y, z, t) = -\frac{p}{q}, \tag{3.3}$$

where

$$\begin{aligned} p &= 2\left(2a_1\left(a_4 t + a_1 x + a_2 y - a_2 z + a_5\right) + 2a_6\left(a_9 t + a_6 x - a_8 y + a_8 z + a_{10}\right)\right), \\ q &= \left(a_4 t + a_1 x + a_2 y - a_2 z + a_5\right)^2 + \left(a_9 t + a_6 x - a_8 y + a_8 z + a_{10}\right)^2 + a_{11}. \end{aligned}$$

The evolution 3D-figures are discussed when $z = 1$ as Figure 1, the lump solution is propagated at the oblique direction.



(a) $t=-5$ (b) $t=0$ (c) $t=5$

Figure 1. Evolution 3D-figures when $z=1$ corresponding to $a_1=8, a_2=10, a_4=-8, a_5=2, a_6=1, a_8=20, a_9=6, a_{10}=1, a_{11}=1$.

Case (2):

$$\begin{aligned} &a_1=a_1, a_2=-\frac{a_1a_8}{a_6}, a_3=\frac{a_1a_8}{a_6}, a_4=\frac{a_1a_9}{a_6}, a_5=a_5,\\ &a_6=a_6, a_7=-a_8, a_8=a_8, a_9=a_9, a_{10}=a_{10}, a_{11}=a_{11}, \end{aligned} \tag{3.4}$$

when $a_6\neq 0$, the corresponding solution to BLMP-like equation is as follows:

$$u(x,y,z,t)=-\frac{p}{q}, \tag{3.5}$$

where

$$\begin{aligned} p&=4\left(a_1^2\left(a_9t+a_6x+a_8(z-y)\right)+a_6^2\left(a_9t+a_6x+a_8(z-y)+a_{10}\right)+a_5a_6a_1\right),\\ q&=a_6\left(\frac{\left(a_1\left(a_9t+a_6x+a_8(z-y)\right)+a_5a_6\right)^2}{a_6^2}\right.\\ &\qquad\left.+\left(a_9t+a_6x+a_8(z-y)+a_{10}\right)^2+a_{11}\right). \end{aligned}$$

Case (3):

$$\begin{aligned} &a_1=a_1, a_2=a_2, a_3=-a_2, a_4=-\frac{a_1a_9}{a_6}, a_5=a_5,\\ &a_6=a_6, a_7=-a_8, a_8=a_8, a_9=0, a_{10}=a_{10}, a_{11}=a_{11}, \end{aligned} \tag{3.6}$$

when $a_6\neq 0$, the corresponding solution to BLMP-like equation is as follows:

$$u(x,y,z,t)=-\frac{p}{q}, \tag{3.7}$$

where

$$\begin{aligned} p=2&\left(2a_1\left(a_1\left(\frac{a_9t}{a_6}+x\right)+a_2(y-z)+a_5\right)\right.\\ &\left.+2a_6\left(a_9t+a_6x+a_8(z-y)+a_{10}\right)\right), \end{aligned}$$

$$
\begin{aligned}
q = &\left(a_1\left(\frac{a_9 t}{a_6}+x\right)+a_2(y-z)+a_5\right)^2 \\
&+\left(a_9 t+a_6 x+a_8(z-y)+a_{10}\right)^2+a_{11}.
\end{aligned}
$$

4. Interaction phenomenons between lump and solition solutions

Assuming

$$
\begin{aligned}
&f=g^2+h^2+\cosh(\xi)+a_{16}, \\
&g=a_1 x+a_2 y+a_3 z+a_4 t+a_5, \\
&h=a_6 x+a_7 y+a_8 z+a_9 z+a_{10}, \\
&\xi=a_{11} x+a_{12} y+a_{13} z+a_{14} t+a_{15},
\end{aligned} \tag{4.1}
$$

where $a_i(i=1,2,...16)$ are real constants determine later. With the help of Mathematic, substituting (4.1) into Equation (2.1), solving the determining equations of $a_i(i=1,2,...16)$, we obtain the following seven-kind interaction solutions of the BLMP-like equation.

Case (1):

$$
\begin{aligned}
&a_1=a_1, a_2=-a_4, a_3=a_3, a_4=a_4, a_5=a_5, a_6=a_6, a_7=-a_9, a_8=a_8, a_9=a_9, \\
&a_{10}=a_{10}, a_{11}=a_{11}, a_{12}=-a_{14}, a_{13}=a_{13}, a_{14}=a_{14}, a_{15}=a_{15}, a_{16}=a_{16},
\end{aligned} \tag{4.2}
$$

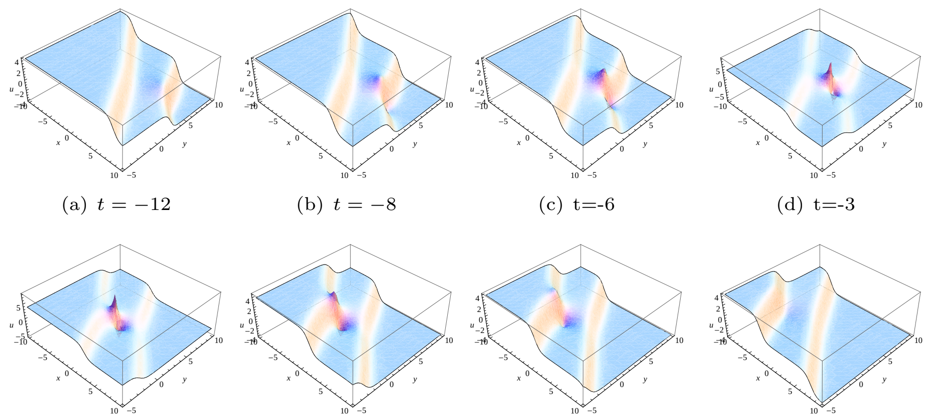
where $a_i(i=1,3,4,5,6,8,9,10,11,13,14,15,16)$ are arbitrary constants, the corresponding solution to BLMP-like equation is as follows:

$$
u(x,y,z,t)=-\frac{p}{q}, \tag{4.3}
$$

where

$$
\begin{aligned}
p = &\, 2\left(2a_1\left(a_3 t+a_1 x-a_4 y+a_4 z+a_5\right)+2a_6\left(a_8 t+a_6 x-a_9 y+a_9 z+a_{10}\right)\right. \\
&\left.+a_{11}\sinh\left(a_{13} t+a_{11} x-a_{14} y+a_{14} z+a_{15}\right)\right), \\
q = &\left(a_3 t+a_1 x-a_4 y+a_4 z+a_5\right)^2+\left(a_8 t+a_6 x-a_9 y+a_9 z+a_{10}\right)^2 \\
&+\cosh\left(a_{13} t+a_{11} x-a_{14} y+a_{14} z+a_{15}\right)+a_{16}.
\end{aligned}
$$

The evolution 3D-figures of a special interaction phenomenon are given along with the time, when $z=2$ in Figure 2.



(a) $t=-12$ (b) $t=-8$ (c) t=-6 (d) t=-3

(e) $t=0$ (f) $t=2$ (g) t=4 (h) t=10

Figure 2. Evolution 3D-figures of lump-solition phenomenon when $z=2$ corresponding to $a_1=5, a_3=6, a_4=-8, a_5=2, a_6=6, a_8=2, a_9=10, a_{10}=1, a_{11}=-2, a_{13}=-1, a_{14}=2, a_{15}=1, a_{16}=1$

Case (2)

$$a_1=a_1, a_2=0, a_3=-3a_1a_{11}^2, a_4=0, a_5=a_5, a_6=a_6, a_7=a_7, a_8=a_8, a_{10}=a_{10},$$
$$a_9=-a_7, a_{11}=a_{11}, a_{12}=-a_{14}, a_{13}=0, a_{14}=a_{14}, a_{15}=a_{15}, a_{16}=a_{16},$$

where $a_i(i=1,5,6,7,8,10,11,14,15,16)$ are arbitrary constants, the corresponding solution to BLMP-like equation is as follows:

$$u(x,y,z,t)=-\frac{p}{q},$$

where

$$\begin{aligned} p=\;&2\left(2a_6\left(a_8t+a_6x+a_7(y-z)+a_{10}\right)+2a_1\left(a_1\left(x-3a_{11}^2t\right)+a_5\right)\right.\\ &\left.+a_{11}\sinh\left(a_{11}x+a_{14}(z-y)+a_{15}\right)\right),\\ q=\;&\left(a_8t+a_6x+a_7(y-z)+a_{10}\right)^2+\left(a_1\left(x-3a_{11}^2t\right)+a_5\right)^2\\ &+\cosh\left(a_{11}x+a_{14}(z-y)+a_{15}\right)+a_{16}. \end{aligned}$$

Case (3)

$$a_1=a_1, a_2=0, a_3=a_3, a_4=0, a_5=a_5, a_6=a_6, a_7=a_7, a_8=a_8, a_9=-a_7,$$
$$a_{10}=a_{10}, a_{11}=a_{11}, a_{12}=-a_{14}, a_{13}=0, a_{14}=a_{14}, a_{15}=a_{15}, a_{16}=a_{16},$$

where $a_i(i=1,5,6,7,8,10,11,14,15,16)$ are arbitrary constants, the corresponding solution to BLMP-like equation is as follows:

$$u(x,y,z,t)=-\frac{p}{q},$$

$$\begin{aligned} p=\;&2\left(2a_6\left(a_8t+a_6x+a_7(y-z)+a_{10}\right)+2a_1\left(a_3t+a_1x+a_5\right)\right.\\ &\left.+a_{11}\sinh\left(a_{11}x+a_{14}(z-y)+a_{15}\right)\right), \end{aligned}$$

$$q = (a_8 t + a_6 x + a_7(y-z) + a_{10})^2 + (a_3 t + a_1 x + a_5)^2 \\ + \cosh(a_{11}x + a_{14}(z-y) + a_{15}) + a_{16}.$$

Case (4)

$$a_1 = a_1, a_2 = a_2, a_3 = a_3, a_4 = -a_2, a_5 = a_5, a_6 = a_6, a_7 = -a_9, a_8 = a_8, a_9 = a_9, \\ a_{10} = a_{10}, a_{11} = a_{11}, a_{12} = -a_{14}, a_{13} = 0, a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = a_{16},$$

where $a_i (i = 1, 2, 3, 5, 6, 8, 9, 10, 11, 14, 15, 16)$ are arbitrary constants, the corresponding solution to BLMP-like equation is as follows:

$$u(x, y, z, t) = -\frac{p}{q},$$

$$p = 2\left(2a_1(a_3 t + a_1 x + a_2(y-z) + a_5) + 2a_6(a_8 t + a_6 x - a_9 y + a_9 z + a_{10}) \right. \\ \left. + a_{11}\sinh(a_{11}x + a_{14}(z-y) + a_{15})\right),$$
$$q = (a_3 t + a_1 x + a_2(y-z) + a_5)^2 + (a_8 t + a_6 x - a_9 y + a_9 z + a_{10})^2 \\ + \cosh(a_{11}x + a_{14}(z-y) + a_{15}) + a_{16}.$$

Case (5)

$$a_1 = a_1, a_2 = -\frac{a_3 a_9}{a_8}, a_3 = a_3, a_4 = \frac{a_3 a_9}{a_8}, a_5 = a_5, a_6 = \frac{a_1 a_8}{a_3}, a_7 = -a_9, a_8 = a_8, \\ a_9 = a_9, a_{10} = a_{10}, a_{11} = 0, a_{12} = -a_{14}, a_{13} = 0, a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = a_{16},$$

where $a_i (i = 1, 3, 5, 8, 9, 10, 14, 15, 16)$ are arbitrary constants, when $a_3 \neq 0$ and $a_8 \neq 0$ the corresponding solution to BLMP-like equation is as follows:

$$u(x, y, z, t) = -\frac{p}{q},$$

where

$$p = 4a_1\left(\frac{a_8(a_3(a_8 t + a_9(z-y) + a_{10}) + a_1 a_8 x)}{a_3^2} + a_3 t + a_1 x - \frac{a_3 a_9 y}{a_8} + \frac{a_3 a_9 z}{a_8} + a_5\right),$$
$$q = \frac{(a_3(a_8 t + a_9(z-y)) + a_1 a_8 x + a_5 a_8)^2}{a_8^2} \\ + \left(a_8\left(\frac{a_1 x}{a_3} + t\right) + a_9(z-y) + a_{10}\right)^2 + \cosh(a_{14}(y-z) - a_{15}) + a_{16}.$$

Case (6)

$$a_1 = a_1, a_2 = -a_4, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = \frac{a_1 a_8}{a_3}, a_7 = -a_9, a_8 = a_8, \\ a_9 = a_9, a_{10} = a_{10}, a_{11} = 0, a_{12} = -a_{14}, a_{13} = 0, a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = a_{16},$$

where $a_i (i = 1, 3, 4, 5, 8, 9, 10, 14, 15, 16)$ are arbitrary constants, when $a_3 \neq 0$ the corresponding solution to BLMP-like equation is as follows:

$$u(x, y, z, t) = -\frac{p}{q},$$

where

$$p = 4a_1\left(\frac{a_8\left(a_3\left(a_8t + a_9(z-y) + a_{10}\right) + a_1a_8x\right)}{a_3^2} + a_3t + a_1x - a_4y + a_4z + a_5\right),$$

$$q = \left(a_3t + a_1x - a_4y + a_4z + a_5\right)^2 + \left(a_8\left(\frac{a_1x}{a_3} + t\right) + a_9(z-y) + a_{10}\right)^2$$
$$+ \cosh\left(a_{14}(y-z) - a_{15}\right) + a_{16}.$$

Case (7)

$$a_1 = a_1, a_2 = -a_4, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = a_6, a_7 = -a_9, a_8 = a_8, a_9 = a_9,$$
$$a_{10} = a_{10}, a_{11} = 0, a_{12} = -a_{14}, a_{13} = 0, a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = a_{16},$$

where $a_i(i = 1, 3, 4, 5, 6, 8, 9, 10, 14, 15, 16)$ are arbitrary constants, the corresponding solution to BLMP-like equation is as follows:

$$u(x, y, z, t) = -\frac{p}{q},$$

where

$$p = 2\left(2a_1\left(a_3t + a_1x - a_4y + a_4z + a_5\right) + 2a_6\left(a_8t + a_6x - a_9y + a_9z + a_{10}\right)\right),$$
$$q = \left(a_3t + a_1x - a_4y + a_4z + a_5\right)^2 + \left(a_8t + a_6x - a_9y + a_9z + a_{10}\right)^2$$
$$+ \cosh\left(a_{14}(y-z) - a_{15}\right) + a_{16}.$$

5. Breather-soliton solution

To search for the breather-soliton solution of the Boiti-Leon-Manna-Pempinelli-like equation, we would like to start from an ansatz

$$\begin{aligned} f &= e^{-g} + e^g + \cos(h) + a_{11}, \\ g &= a_1x + a_2y + a_3t + a_4z + a_5, \\ h &= a_6x + a_7y + a_8t + a_9z + a_{10}. \end{aligned} \tag{5.1}$$

where $a_i(i = 1, 2, \cdots, 11)$ are real constants determined later. With the help of mathematics, by substituting (5.1) into Equation (2.1), solving the determining equations of $a_i(i = 1, 2, \cdots, 11)$, we obtain the following two kinds of solution as follows:

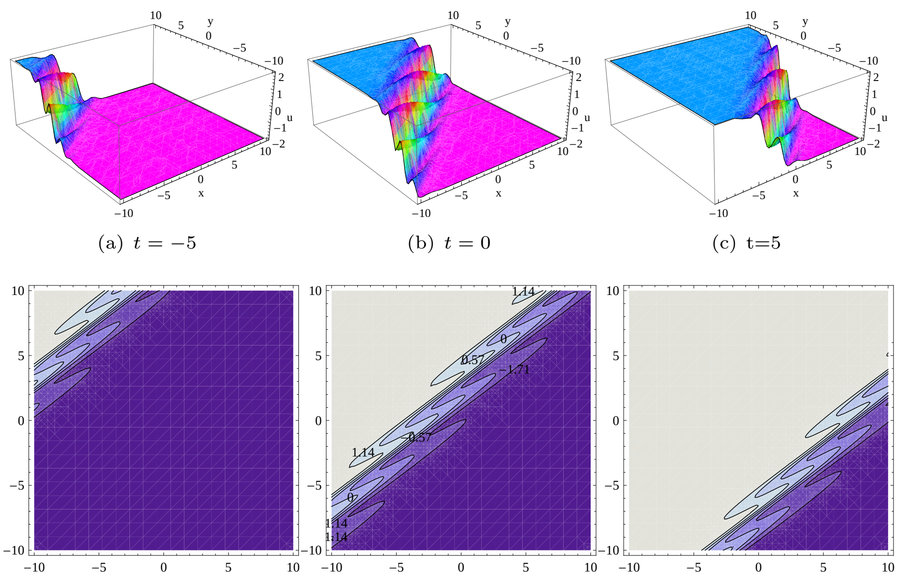
Case (1):

$$a_1 = a_1, a_2 = -a_4, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = a_6,$$
$$a_7 = -a_9, a_8 = a_8, a_9 = a_9, a_{10} = a_{10}, a_{11} = a_{11},$$

where $a_i(i = 1, 3, 4, 5, 6, 8, 9, 10, 11)$ are arbitrary constants, the corresponding solution to BLMP-like equation is as follows:

$$u(x, y, z, t) = \frac{\xi}{\eta},$$
$$\xi = 2a_6\sin\left(a_6x - a_9y + a_9z + a_{10}\right),$$
$$\eta = \cos\left(a_6x - a_9y + a_9z + a_{10}\right) + e^{a_4y - a_4z - a_5} + e^{a_4(-y) + a_4z + a_5} + a_{11}.$$

The evolution 3D-figures and contour plots of a special breather-soliton solution are given along with the time, when $z = 1$ in Figure 3.



(a) $t = -5$ (b) $t = 0$ (c) t=5

(d) $t = -5$ (e) $t = 0$ (f) $t = 5$

Figure 3. The evolution 3D-figures and contour plots when $z = 1$ corresponding to $a_1 = 1, a_3 = -1, a_4 = 1, a_5 = 2, a_6 = 2, a_8 = 2, a_9 = 3, a_{10} = 3, a_{11} = 3$

Case (2)

$$\begin{aligned} &a_1 = 0, a_2 = -a_4, a_3 = 0, a_4 = a_4, a_5 = a_5, a_6 = a_6, \\ &a_7 = -a_9, a_8 = a_8, a_9 = a_9, a_{10} = a_{10}, a_{11} = a_{11}, \end{aligned}$$

where $a_i (i = 4, 5, 6, 8, 9, 10, 11)$ are arbitrary constants, the corresponding solution to BLMP-like equation is as follows:

$$u(x, y, z, t) = \frac{2a_6 \sin(a_6 x - a_9 y + a_9 z + a_{10})}{\cos(a_6 x - a_9 y + a_9 z + a_{10}) + e^{a_4(y-z)-a_5} + e^{a_4(z-y)+a_5} + a_{11}}.$$

6. Discussion and conclusions

Based on the generalized bilinear operators (1.1), a (3+1)-dimensional Boiti-Leon-Manna-Pempinelli-like equation was introduced. Through discussing lump solutions of the BLMP-like equation, we found three kinds of lump solutions. From Figure 1, we can see that one special lump solution moves at the oblique direction.

Through discussing lump-soliton interaction phenomenon, we obtain seven-kind interaction solutions. From one special solution, we can see the interaction phenomenon is appeared and disappeared along with the time as Figure 2. Through discussing the breather-solition, we get two-kinds breather-soliton solution. From Figure 3, we can see a kink-breather soliton moved in oblique direction.

References

[1] G. Adomian, *A review of the decomposition method in applied mathematics*, Journal Mathematical Analysis and Applications, 1988, 135, 501–544.

[2] S. Batwa and W. Ma, *Lump Solutions to a (2+1)-Dimensional Fifth-Order KdV-Like Equation*, Advances in Mathematical Physics, 2018, Article ID 2062398, 6 pages.

[3] G. W. Bluman and S. Kumei, *Symmetries and Differential Equations*, World Publishing Corporation, Springer-Verlag, Heidelberg, 1993.

[4] S. Chen and W. Ma, *Lump solutions to a generalized Bogoyavlensky-Konopelchenko equation*, Frontiers of Mathematics in China, 2018, 13, 525–534.

[5] M. T. Darvisi, M. Najafi, L. Kavitha and M. Venkatesh, *Stair and Step Soliton Solutions of the Integrable (2+1) and (3+1)-Dimensional Boiti-Leon-Manna-Pempinelli Equations*, Communications in Theoretical Physics, 2012, 58, 785–794.

[6] Y. Du and Y. Yun, *Rational solutions to two Sawada-Kotera-like equations*, Modern Physics Letters B, 2019, Article ID 1950108, 9 pages.

[7] Y. Du and Y. Yun, *Rational solutions to a generalized (2+1)-dimensional Shallow-Water-Wave-like equation*, Journal of Mathematics, 2019, 39, 915–920.

[8] E. Fan, *Extended tanh-function method and its applications to nonlinear equations*, Physics Letters A, 2000, 277, 212–220.

[9] K. A. Gorshkov, D. E. Pelinovsky and Y. A. Stepanyants, *Normal and anomalous scattering, formation and decay of bound states of two-dimensional solitons described by the Kadomtsev-Petviashvili equation*, Journal of Experimental and Theoretical Physics, 1993, 104, 2704–2720.

[10] J. He, *A Coupling Method of a Homotopy Technique and a Perturbation Technique for Non-Linear Problems*, International Journal of Nonlinear Mechanics, 2000, 35(1), 37–43.

[11] J. He, *An elementary introduction to the homotopy perturbation method*, Computers & Mathematics with Applications, 2009, 57, 410–412.

[12] R. Hirota, *The Direct Method in Soliton Theory*, Cambridge University Press, Cambridge, 2004.

[13] S. Liao, *The proposed homotopy analysis technique for the solution of nonlinear problems*, Ph.D. Thesis, Shanghai Jiao Tong University, Shanghai, 1992.

[14] X. Lv, *A note on rational solutions to a Hirota-Satsuma-like equation*, Applied Mathematics Letters, 2016, 58, 13–18.

[15] J. Lv and S. Bilige, *Diversity of interaction solutions to the (3+1)-dimensional Kadomtsev-Petviashvili-Boussinesq-like equation*, Modern Physics Letters B, 2018, 32(26), Article ID 1850311.

[16] X. Lv and W. Ma, *Rational solutions to an extended Kadomtsev-Petviashvili-like equation with symbolic computation*, Computers & Mathematics with Applications, 2016, 71, 1560–1567.

[17] W. Ma, *Generalized bilinear differential equations*, Studies in Nonlinear Sciences, 2011, 2(4), 140–144.

[18] W. Ma, *Bilinear equations,Bell polynomials and linear superposition principal*, Journal of Physics Conference, 2013, 411, Article ID 12021.

[19] W. Ma, *Bilinear Equations and Resonant Solutions Characterized by Bell Polynomials*, Reports on Mathematical Physics, 2013, 72, 41–56.

[20] W. Ma, *Lump solutions to the Kadomtsev-Petviashvili equation*, Physics Letters A, 2015, 379, 1975–1978.

[21] W. Ma, *RiemannCHilbert problems and N-soliton solutions for a coupled mKdV system*, Journal Geometry and Physics, 2018, 132, 45–54.

[22] W. Ma, *Lump and interaction solutions to linear PDEs in 2+1 dimensions via symbolic computation*, Modern Physics Letters B, 2019, 33(36), Article ID 1950457.

[23] W. Ma and L. Zhang, *Lump solutions with higher-order rational dispersion relations*, Pramana-Journal of Physics, 2020, 94, 43.

[24] W. Ma, Y. Zhang and Y. Tang, *Symbolic Computation of Lump Solutions to a Combined Equation Involving Three Types of Nonlinear Terms*, East Asian Journal on Applied Mathematics, 2020, 10(4), 732–745.

[25] W. Ma and Y. Zhou, *Lump solutions to nonlinear partial differential equations via Hirota bilinear forms*, Journal of Differential Equations, 2018, 264(4), 2633–2659.

[26] S. V. Manakov, V. E. Zakhorov and L. A. Bordag, *Two-dimensional solitons of the Kadomtsev-Petviashvili equation and their interaction*, Physics Letters A, 1977, 63, 205–206.

[27] R. Rach, *A new definition of the Adomian polynomials*, Kybernetes, 2008, 37, 910–955.

[28] N. Singh and Y. Stepanyants, *Obliquely propagating skew KP lumps*, Wave Motion: An International Journal Reporting Research, 2016, 64, 92–102.

[29] J. Yang and W. Ma, *Determining lump solutions for a combined soliton equation in (2+1)-dimensions*, European Physical Journal Plus, 2020, 135(6), 494.

[30] J. Yang, W. Ma and Z. Qin, *Lump and lump-soliton solutions to the (2+1)-dimensional Ito equation*, Analysis and Mathematical Physics, 2018, 8, 427–436.

[31] J. Yu and Y. Sun, *Study of lump solutions to dimensionally reduced generalized KP equations*, Nonlinear Dynamics, 2017, 87, 2755–2763.

[32] Y. Yun and T. Chaolu, *A Further Improved Tanh Method and New Traveling Wave Solutions of 2D-KdV Equation*, Journal of Inner Mongolia Univeisity, 2011, 42, 604–609.

[33] Y. Yun and T. Chaolu, *Homotopy Perturbation Method For Viscous Heating In Plane Couette Flow*, Thermal Science, 2013, 17, 1355–1360.

[34] Y. Yun and T. Chaolu, *Application of the homotopy perturbation method for the large deflection problem of a circular plate*, Applied Mathematical Modelling, 2015, 39, 1308–1316.

[35] Y. Yun and T. Chaolu, *Classical and nonclassical symmetry classifications of nonlinear wave equation with dissipation*, Applied Mathematics and Mechanics, 2015, 36(3), 365–378.

[36] R. Zhang and S. D. Bilige, *New periodic wave, cross-kink wave and the interaction phenomenon for the Jimbo-Miwa-like equation*, Computers & Mathematics with Applications, 2019, 78(3), 754–764.

[37] Y. Zhang and W. Ma, *Rational solutions to a KdV-like equation*, Computers & Mathematics with Applications, 2015, 256, 252–256.