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Explicit Traveling Wave Solutions and Their Dynamical Behaviors for the Coupled Higgs Field Equation*

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Abstract In this paper, we focus on the traveling wave solutions of the coupled Higgs field equation from the perspective of dynamical systems. Through the phase portraits, in addition to periodic wave solutions and solitary wave solutions, we also obtain explicit periodic singular wave solutions, singular wave solutions and kink wave solutions, which were not found in the previous works. The dynamical behavior of these solutions and their internal relations are revealed through asymptotic analysis. The results will help supplement the study of field equation.

Keywords Coupled Higgs field equation, Traveling wave solutions, Kink wave solutions.

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1. Introduction

The classical Higgs equation [16]

$$u_{tt} - u_{xx} - \beta u + \gamma |u|^2 u = 0 \tag{1.1}$$

has important applications in particle physics, field theory and electromagnetic waves [4]. Equation (1.1) is attributed to the classical ϕ_4 field theory in physics of elementary particles and fields. As a generalized form of equation (1.1), the coupled Higgs field equation

$$\begin{cases} u_{tt} - u_{xx} - \beta u + \gamma |u|^2 u - 2uv = 0, \\ v_{tt} + v_{xx} - \gamma (|u|^2)_{xx} = 0 \end{cases} \tag{1.2}$$

has attracted considerate attention [1,4–8,13–15,18,24,25]. Equation (1.2) describes a system of conserved scalar nucleons interacting with neutral scalar mesons in particle physics. Here, the function $v = v(x,t)$ indicates a real scalar meson field,

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and $u = u(x,t)$ stands for a complex scalar nucleon field. The subscripts t, x of u and v denote appropriate partial derivatives with respect to the time and space variables.

There have been many works on the solutions of equation (1.2) from various aspects. Tajiri [13] derived the N-soliton solution of equation (1.2) exploiting Hirota bilinear method. Later, by Hirota bilinear method, Hu et al., [6] looked for homoclinic solution of equation (1.2). The authors in [4, 14, 24] obtained the bright soliton, periodic wave and doubly periodic wave solutions of equation (1.2). Zha et al., [8, 25] studied the first-order rogue wave solution of equation (1.2). Hon and Fan [5] used an algebraic method to construct solitary wave solutions, Jacobi periodic wave solutions and a range of other solutions of physical interest. Wazwaz and his co-author [15, 18] obtained a variety of exact periodic waves and solitary wave solutions of equation (1.2). Jabbri et al., [7] combined He's semi-inverse and (G'/G)-expansion methods to construct the exact solutions of equation (1.2). Ali et al., [1] found a variety of solitary wave solutions by using rational $\exp(-\varphi(\eta))$-expansion method.

Despite the success of these attempts in understanding solutions of equation (1.2), we note that the above works did not find kink waves of equation (1.2), and the dynamical behavior of the solutions and their internal relations are not so clear. Therefore, we intend to study equation (1.2) from the viewpoint of geometry. More precisely, we exploit qualitative theory of differential equations and bifurcation method of dynamical systems [2, 3, 9–12, 17, 19–23, 26] to study the traveling wave solutions of equation (1.2) and to reveal their dynamical behavior and inside relations. Through analyzing the phase portrait, in addition to periodic wave solutions and solitary wave solutions, we also obtain explicit periodic singular wave solutions, singular wave solutions and kink wave solutions, which were not found in the above works. The dynamical behavior of these solutions and their internal relations are uncovered through asymptotic analysis.

2. Qualitative analysis and phase portraits

To study the traveling wave solutions of equation (1.2), assume

$$u(x,t) = e^{i\eta}\varphi(\xi), v(x,t) = \phi(\xi), \eta = px + rt, \xi = kx + dt, \tag{2.1}$$

where $\varphi(\xi)$ and $\phi(\xi)$ are real functions, and p, r, k and d are real constants.

Substituting (2.1) into equation (1.2), we have

$$\begin{cases} (d^2 - k^2)\varphi'' - (r^2 - p^2 + \beta)\varphi + \gamma\varphi^3 - 2\varphi\phi = 0, \\ (d^2 + k^2)\phi'' - \gamma k^2(\varphi^2)'' = 0, \\ rd = kp. \end{cases} \tag{2.2}$$

Integrating the second equation of (2.2) twice and letting the first integral constant be zero, we have

$$\phi = \frac{\gamma k^2 \varphi^2}{d^2 + k^2} + g, \tag{2.3}$$

where g is an integral constant.

Substituting (2.3) into the first equation of (2.2), we have

$$\varphi'' - n\varphi - m\varphi^3 = 0, \tag{2.4}$$

where $m = -\frac{\gamma}{d^2+k^2}$, and $n = \frac{r^2-p^2+\beta+2g}{d^2-k^2}$ with $d^2 \neq k^2$.

Letting $\varphi' = y$, we obtain the following planar system

$$\begin{cases} \frac{d\varphi}{d\xi} = y, \\ \frac{dy}{d\xi} = m\varphi^3 + n\varphi. \end{cases} \tag{2.5}$$

Obviously, the above system (2.5) is a Hamiltonian system with Hamiltonian function

$$H(\varphi, y) = y^2 - \frac{1}{2}m\varphi^4 - n\varphi^2. \tag{2.6}$$

To investigate phase portraits of (2.5), set

$$f(\varphi) = m\varphi^3 + n\varphi. \tag{2.7}$$

Obviously, when $mn \geq 0$ ($m \neq 0$), $f(\varphi)$ has only one zero $\varphi_0 = 0$. When $mn < 0$, $f(\varphi)$ has three zeros,

$$\varphi_- = -\sqrt{-\frac{n}{m}}, \varphi_0 = 0, \text{ and } \varphi_+ = \sqrt{-\frac{n}{m}}. \tag{2.8}$$

If $(\varphi, 0)$ is one of the singular points of system (2.5), then the characteristic values of the linearized system of system (2.5) at the singular point $(\varphi, 0)$ are

$$\lambda_\pm = \pm\sqrt{f'(\varphi)}. \tag{2.9}$$

According to the qualitative theory of dynamical systems, we obtain the phase portraits of system (2.5) in Figure 1.

3. Traveling wave solutions of equation (1.2), and their dynamical behavior and internal relations

To state conveniently, set

$$h^* = |H(\varphi_+, 0)| = |H(\varphi_-, 0)| = \frac{n^2}{2|m|}. \tag{3.1}$$

Then, we have the following results.

Proposition 1. For the cases when $m > 0$ and $n < 0$, we have

(i) when $h = h^*$, system (2.5) has a pair of kink (anti-kink) wave solutions

$$\varphi_1(\xi) = \pm\sqrt{-\frac{n}{m}} \tanh \sqrt{-\frac{n}{2}}\xi \tag{3.2}$$

and two singular wave solutions

$$\varphi_2(\xi) = \pm\sqrt{-\frac{n}{m}} \coth \sqrt{-\frac{n}{2}}\xi; \tag{3.3}$$

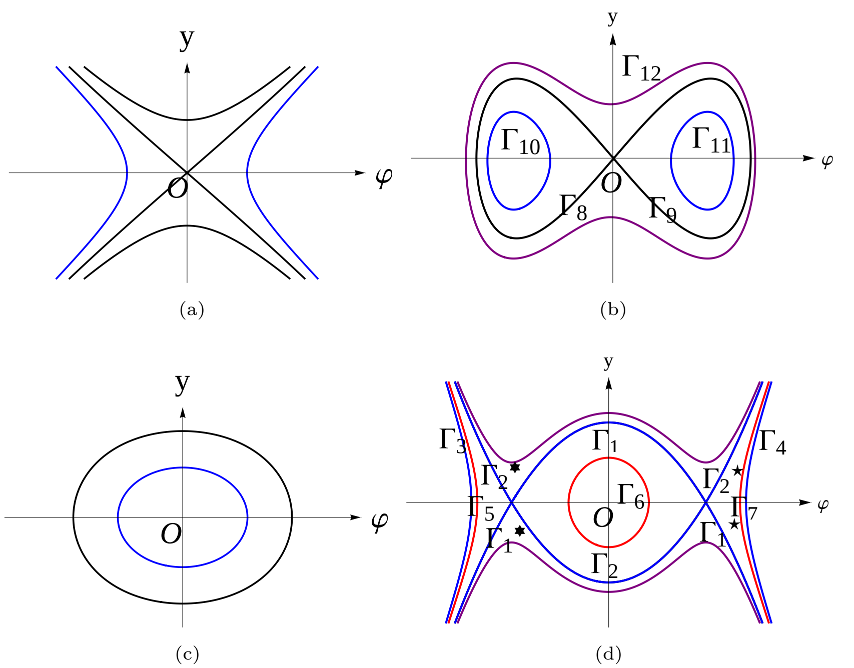

Figure 1. The phase portraits of system (2.5) when (a) $m > 0, n \geq 0$; (b) $m < 0, n > 0$; (c) $m < 0, n \leq 0$; (d) $m > 0, n < 0$

(ii) when $h = 0$, system (2.5) has four periodic singular wave solutions

$$\varphi_3(\xi) = \pm\sqrt{-\frac{2n}{m}} \csc \sqrt{-n}\xi, \tag{3.4}$$

$$\varphi_4(\xi) = \pm\sqrt{-\frac{2n}{m}} \sec \sqrt{-n}\xi; \tag{3.5}$$

(iii) when $0 < h < h^*$, system (2.5) has a family of periodic singular wave solutions

$$\varphi_5(\xi) = \pm\frac{\alpha_2}{sn(\alpha_2\sqrt{\frac{m}{2}}\xi, \frac{\alpha_1}{\alpha_2})} \tag{3.6}$$

and a family of periodic wave solutions

$$\varphi_6(\xi) = \pm\alpha_1 sn(\alpha_2\sqrt{\frac{m}{2}}\xi, \frac{\alpha_1}{\alpha_2}), \tag{3.7}$$

where $\alpha_1 = \sqrt{\frac{-n-\sqrt{n^2-2mh}}{m}}$, $\alpha_2 = \sqrt{\frac{-n+\sqrt{n^2-2mh}}{m}}$, and $0 < h < h^*$.

Additionally, we have the limit forms,

(1) when $h \to h^* - 0$, the periodic wave solutions $\varphi_6(\xi)$ in (3.7) converge to the pair of kink (antikink) wave solutions $\varphi_1(\xi)$ in (3.2), and the periodic singular wave solutions $\varphi_5(\xi)$ in (3.6) converge to the singular wave solutions $\varphi_2(\xi)$ in (3.3) respectively;

(2) when $h \to 0+0$, the periodic singular wave solutions $\varphi_5(\xi)$ in (3.6) converge to the singular wave solutions $\varphi_3(\xi)$ in (3.4) respectively.

Proof. When $m > 0$ and $n < 0$, we consider the following three types of orbits of system (2.5) in Figure 1(d).

(i) Two heteroclinic orbits Γ_1 and Γ_2 connected at saddles $(\varphi_-, 0)$, and $(\varphi_+, 0)$ can be expressed as

$$y = \pm\sqrt{\frac{m}{2}}\sqrt{(\varphi-\varphi_-)^2(\varphi-\varphi_+)^2}. \tag{3.8}$$

Substituting (3.8) into $\frac{d\varphi}{d\xi} = y$ and integrating them along the heteroclinic orbits Γ_1 and Γ_2, it follows that

$$\int_0^{\varphi} \frac{1}{(s-\varphi_-)(\varphi_+-s)}ds = \pm\sqrt{\frac{m}{2}}\int_0^{\xi} ds, \tag{3.9}$$

$$\int_{\varphi}^{+\infty} \frac{1}{(s-\varphi_-)(s-\varphi_+)}ds = \pm\sqrt{\frac{m}{2}}\int_0^{\xi} ds. \tag{3.10}$$

Completing the above integrals, we obtain (3.2) and (3.3) respectively.

(ii) The two special orbits Γ_3 and Γ_4, which have the same Hamiltonian with that of the center point $(0,0)$, can be expressed as

$$y = \pm\sqrt{\frac{m}{2}}\varphi\sqrt{\left(\varphi-\sqrt{-\frac{2n}{m}}\right)\left(\varphi+\sqrt{-\frac{2n}{m}}\right)}. \tag{3.11}$$

Substituting (3.11) into $\frac{d\varphi}{d\xi} = y$ and integrating them along the two orbits Γ_3 and Γ_4, it follows that

$$\int_{\varphi}^{\infty} \frac{1}{s\sqrt{\left(s-\sqrt{-\frac{2n}{m}}\right)\left(s+\sqrt{-\frac{2n}{m}}\right)}}ds = \pm\sqrt{\frac{m}{2}}\int_0^{\xi} ds. \tag{3.12}$$

Completing the above integrals, we obtain (3.4).
Further, note that if $\varphi = \varphi(\xi)$ is a solution of system (2.5), then $\varphi = \varphi(\xi+\gamma)$ is also a solution of system (2.5). Specially, taking $\gamma = \frac{\pi}{2}$, we obtain another two periodic singular solutions (3.5).

(iii) The three orbits Γ_5, Γ_6 and Γ_7 passing the points $(-\alpha_2, 0)$, $(-\alpha_1, 0)$, $(\alpha_1, 0)$ and $(\alpha_2, 0)$ can be expressed as

$$y = \pm\sqrt{\frac{m}{2}}\sqrt{(\alpha_1^2-\varphi^2)(\alpha_2^2-\varphi^2)}. \tag{3.13}$$

Substituting (3.13) into $\frac{d\varphi}{d\xi} = y$ and integrating them along Γ_5, Γ_6, and Γ_7, we have

$$\int_{\varphi}^{\infty} \frac{1}{\sqrt{(\alpha_1^2-s^2)(\alpha_2^2-s^2)}}ds = \pm\sqrt{\frac{m}{2}}\int_0^{\xi} ds, \tag{3.14}$$

$$\int_0^\varphi \frac{1}{\sqrt{(\alpha_1^2 - s^2)(\alpha_2^2 - s^2)}} ds = \pm\sqrt{\frac{m}{2}} \int_0^\xi ds. \tag{3.15}$$

Completing the above integrals, we obtain (3.6) and (3.7).
Additionally, the limit forms follow easily.

□

Proposition 2. For the cases when $m < 0$ and $n > 0$, we have

(i) when $h = 0$, system (2.5) has two symmetric solitary wave solutions

$$\varphi_7(\xi) = \pm\sqrt{-\frac{2n}{m}} sech\sqrt{n}\xi; \tag{3.16}$$

(ii) when $0 < h < h^*$, system (2.5) has two families of periodic wave solutions

$$\varphi_8(\xi) = \frac{-\alpha_1(\alpha_1 + \alpha_2) + \alpha(\alpha_1 - \alpha_2)(sn(\omega\sqrt{-\frac{m}{2}}\xi, \rho))^2}{\alpha_1 + \alpha_2 + (\alpha_1 - \alpha_2)(sn(\omega\sqrt{-\frac{m}{2}}\xi, \rho))^2}, \tag{3.17}$$

$$\varphi_9(\xi) = \frac{\alpha_1 + \alpha_2 + (\alpha_1 - \alpha_2)(sn(\omega\sqrt{-\frac{m}{2}}\xi, \rho))^2}{\alpha_1(\alpha_1 + \alpha_2) - \alpha_1(\alpha_1 - \alpha_2)(sn(\omega\sqrt{-\frac{m}{2}}\xi, \rho))^2}, \tag{3.18}$$

where $\omega = \frac{\alpha_1+\alpha_2}{2}$ and $\rho = \frac{\alpha_1-\alpha_2}{\alpha_1+\alpha_2}$.

(iii) when $h < 0$, system (2.5) has one family of periodic wave solutions

$$\varphi_{10}(\xi) = -\alpha_1 cn\left(\sqrt{\sqrt{n^2 - 2mh}}\,\xi, \alpha_1\sqrt{-\frac{m}{2\sqrt{n^2 - 2mh}}}\right). \tag{3.19}$$

Additionally, we have the limit forms,

(1) when $h \to 0 - 0$, the periodic wave solutions $\varphi_{10}(\xi)$ in (3.19) converge to the symmetric solitary wave solutions $\varphi_7(\xi)$ in (3.16) respectively;

(2) when $h \to 0+0$, the periodic wave solutions $\varphi_8(\xi)$ in (3.17) and $\varphi_9(\xi)$ in (3.18) converge to the symmetric solitary wave solutions $\varphi_7(\xi)$ in (3.16) respectively.

Proof. When $m < 0$ and $n > 0$, we consider the following two types of orbits of system (2.5) in Figure 1(b).

(i) The two symmetric homoclinic orbits Γ_8 and Γ_9 connected at the saddle $(0, 0)$ can be expressed as

$$y = \pm\sqrt{-\frac{m}{2}}\varphi\sqrt{\left(\sqrt{-\frac{2n}{m}} - \varphi\right)\left(\varphi + \sqrt{-\frac{2n}{m}}\right)}. \tag{3.20}$$

Substituting (3.20) into $\frac{d\varphi}{d\xi} = y$ and integrating them along the two homoclinic orbits Γ_8 and Γ_9, it follows that

$$\int_{-\sqrt{-\frac{2n}{m}}}^\varphi \frac{1}{s\sqrt{\left(\sqrt{-\frac{2n}{m}} - s\right)\left(s + \sqrt{-\frac{2n}{m}}\right)}} ds = \pm\sqrt{-\frac{m}{2}} \int_0^\xi ds, \tag{3.21}$$

$$\int_{\varphi}^{\sqrt{-\frac{2n}{m}}} \frac{1}{s\sqrt{\left(\sqrt{-\frac{2n}{m}}-s\right)\left(s+\sqrt{-\frac{2n}{m}}\right)}}ds = \pm\sqrt{-\frac{m}{2}}\int_0^{\xi} ds. \tag{3.22}$$

Completing the above integrals, we obtain (3.16).

(ii) The two families of periodic orbits Γ_{10} and Γ_{11} can be expressed as

$$y = \pm\sqrt{-\frac{m}{2}}\sqrt{(\varphi^2-\alpha_1^2)(\alpha_2^2-\varphi^2)}. \tag{3.23}$$

Substituting (3.23) into $\frac{d\varphi}{d\xi} = y$ and integrating them along Γ_{10} and Γ_{11}, we have

$$\int_{-\alpha_2}^{\varphi} \frac{1}{\sqrt{(s^2-\alpha_1^2)(\alpha_2^2-s^2)}}ds = \pm\sqrt{-\frac{m}{2}}\int_0^{\xi} ds, \tag{3.24}$$

$$\int_{\varphi}^{\alpha_2} \frac{1}{\sqrt{(s^2-\alpha_1^2)(\alpha_2^2-s^2)}}ds = \pm\sqrt{-\frac{m}{2}}\int_0^{\xi} ds. \tag{3.25}$$

Completing the above integrals, we obtain (3.17) and (3.18).
The family of periodic orbit Γ_{12} can be expressed as

$$y = \pm\sqrt{-\frac{m}{2}}\sqrt{(\alpha_1-\varphi)(\varphi+\alpha_1)(\varphi-c_1)(\varphi-c_1^*)}, \tag{3.26}$$

where $c_1 = i\sqrt{\frac{n-\sqrt{n^2-2mh}}{m}}$, $c_1^* = -i\sqrt{\frac{n-\sqrt{n^2-2mh}}{m}}$ and $h > 0$.
Substituting (3.26) into $\frac{d\varphi}{d\xi} = y$ and integrating them along the orbit Γ_{12}, we have

$$\int_{-\alpha_1}^{\varphi} \frac{1}{\sqrt{(\alpha_1-s)(s+\alpha_1)(s-c_1)(s-c_1^*)}}ds = \pm\sqrt{-\frac{m}{2}}\int_0^{\xi} ds, \tag{3.27}$$

Completing the above integrals, we obtain (3.19).
Additionally, it is easy to obtain the limit forms.

□

To illustrate the limit forms, taking $\beta = -18, \gamma = -10, p = 4, r = 2, k = 1, d = 2, g = 3$, which indicate that $m = 2$, $n = -8$ and $h^* = 16$, we present the process of the periodic wave solution $\varphi_6(\xi)$ tending to the kink wave solution $\varphi_1(\xi)$, when $h \to h^* - 0$ graphically in Figure 2. Additionally, the corresponding graphs of $v(\xi) = \frac{\gamma k^2}{d^2+k^2}\varphi^2(\xi) + g$ are given in Figure 3.

Remark 3.1. From $v(\xi) = \frac{\gamma k^2}{d^2+k^2}\varphi^2(\xi) + g$, note that if $\varphi(\xi)$ is a periodic wave solution with period T, then $v(\xi)$ is a periodic wave solution with period $\frac{T}{2}$. Besides, if $\varphi(\xi)$ is a kink (anti-kink) wave solution, then $v(\xi)$ is a solitary wave solution.

Based on (2.1), (2.2), (2.3), Proposition 1 and Proposition 2, we immediately have the following theorems for equation (1.2).

Theorem 3.1. *When $m > 0$ and $n < 0$, equation (1.2) has the exact solutions in the form of $u(x,t) = e^{i(px+rt)}\varphi(\xi)$ and $v(x,t) = \frac{\gamma k^2}{d^2+k^2}\varphi^2(\xi) + g$ with $rd = kp$, $\xi = kx + dt$ and $\varphi(\xi) = \varphi_i(\xi)$, for $i = 1, \cdots, 6$.*

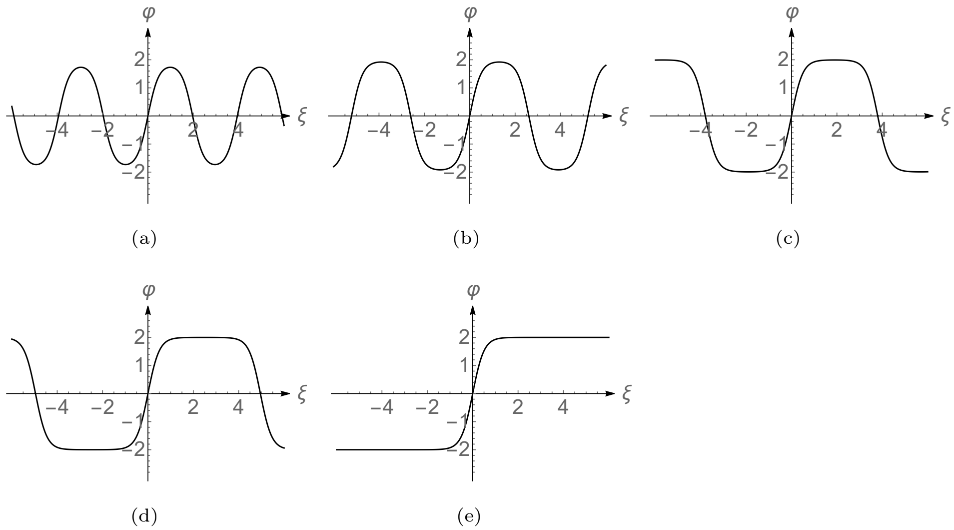

Figure 2. The process of the periodic wave solution $\varphi_6(\xi)$ tending to kink wave solution $\varphi_1(\xi)$ as $h \to h^* - 0$ by taking (a) $h = 15$; (b) $h = 15.9$; (c) $h = 15.999$; (d) $h = 15.99999$; (e) $h = 16$

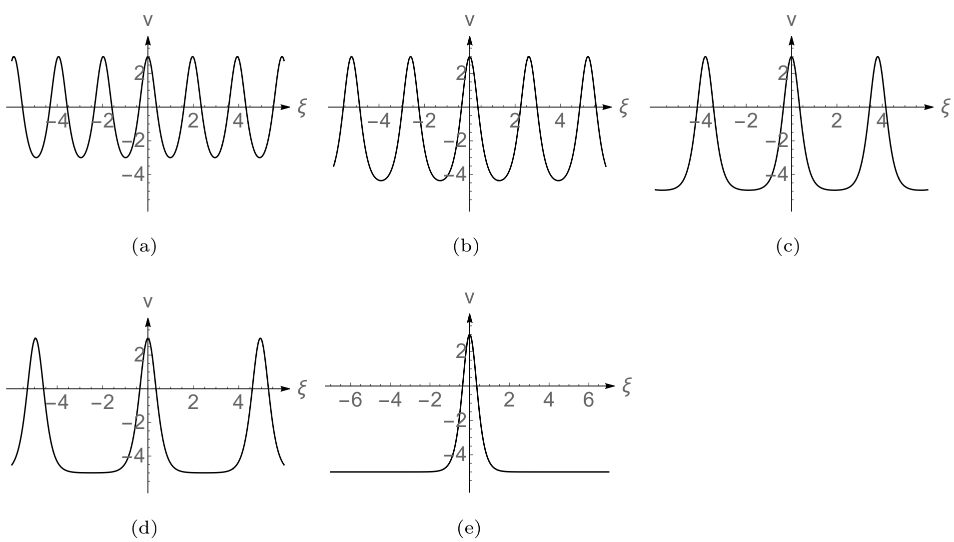

Figure 3. The graphs of $v(\xi) = \frac{\gamma k^2}{d^2+k^2}\varphi^2(\xi) + g$ corresponding to $\varphi(\xi)$ in Figure 2

Theorem 3.2. *When $m < 0$ and $n > 0$, equation (1.2) has the exact solutions in the form of $u(x,t) = e^{i(px+rt)}\varphi(\xi)$ and $v(x,t) = \frac{\gamma k^2}{d^2+k^2}\varphi^2(\xi) + g$ with $rd = kp$, $\xi = kx + dt$ and $\varphi(\xi) = \varphi_i(\xi)$, for $i = 7, \cdots, 10$.*

4. Conclusion

In this paper, we study the traveling wave solutions of equation (1.2) from the perspective of dynamical systems. In addition to traditional periodic wave solutions and solitary wave solutions, we also obtain explicit periodic singular wave solutions, singular wave solutions and kink wave solutions, which were not found in the previous works. We reveal the dynamical behavior of these solutions and their inside relations through asymptotic analysis.

References

[1] A. Ali, M. Iqbal and S. Mohyud-din, *New analytical solutions for nonlinear physical models of the coupled Higgs equation and the Maccari system via rational exp($-\phi(\eta)$)-expansion method*, Pramana–Journal of Physics, 2016, 87(5), 79, 14 pages.

[2] A. Chen, S. Wen, S. Tang, W. Huang and Z. Qiao, *Effects of quadratic singular curves in integrable equations*, Studies in Applied Mathematics, 2015, 134(1), 24–61.

[3] Y. Chen, M. Song and Z. Liu, *Soliton and Riemann theta function quasi-periodic wave solutions for a (2+1)-dimensional generalized shallow water wave equation*, Nonlinear Dynamics, 2015, 82(1–2), 333–347.

[4] E. Fan, K. Chow and J. Li, *On doubly periodic standing wave solutions of the coupled Higgs field equation*, Studies in Applied Mathemtics, 2012, 128(1), 86–105.

[5] Y. Hon and E. Fan, *A series of exact solutions for coupled Higgs field equation and coupled Schrödinger-Boussinesq equation*, Nonlinear Analysis–Theory Methods and Applications, 2009, 71(7–8), 3501–3508.

[6] X. Hu, B. Guo and H. Tam, *Homoclinic orbits for the coupled Schrödinger–Boussinesq equation and coupled Higgs equation*, Journal of the Physical Society of Japan, 2003, 72(1), 189–190.

[7] A. Jabbri, H. Kheiri and A. Bekir, *Exact solutions of the coupled Higgs equation and the Maccari system using He's semi-inverse method and (G'/G)–expansion method*, Computers & Mathematics with Applications, 2011, 62(5), 2177–2186.

[8] G. Mu and Z. Qin, *Rogue waves for coupled Schrödinger–Boussinesq equation and coupled Higgs equation*, Journal of the Physical Society of Japan, 2012, 81(8), Article ID 084001, 6 pages.

[9] C. Pan and Y. Yi, *Some extensions on the soliton solutions for the Novikov equation with cubic nonlinearity*, Journal of Nonlinear Mathematical Physics, 2015, 22(2), 308–320.

[10] L. Shi and Z. Wen, *Several types of periodic wave solutions and their relations of a Fujimoto–Watanabe equation*, Journal of Applied Analysis and Computation, 2019, 9(4), 1193–1203.

[11] M. Song, *Nonlinear wave solutions and their relations for the modified Benjamin–Bona–Mahony equation*, Nonlinear Dynamics, 2015, 80(1–2), 431–446.

[12] M. Song and Z. Liu, *Qualitative analysis and explicit traveling wave solutions for the Davey–Stewartson equation*, Mathematical Methods in the Applied Sciences, 2014, 37(3), 393–401.

[13] M. Tajiri, *On N-soliton solutions of coupled Higgs field equation*, Journal of the Physical Society of Japan, 1983, 52(7), 2277–2280.

[14] B. Talukdar, S. Chosh, A. Saha and D. Pal, *Solutions of the coupled Higgs field equations*, Physical Review E, 2013, 88(1), Article 015201, 4 pages.

[15] H. Triki and A. Wazwaz, *A variety of exact periodic wave and solitary wave solutions for the coupled Higgs equation*, Zeitschrift für Naturforschung Section A–A Journal of Physical Sciences, 2012, 67(10–11), 545–549.

[16] V. Tsegel'nik, *Self-similar solutions of a system of two nonlinear partial differential equations*, Differential Equations, 2000, 36(3), 480–482.

[17] W. Wang, C. Li and W. Zhu, *Bifurcations and single peak solitary wave solutions of an integrable nonlinear wave equation*, Advances in Applied Mathematics and Mechanics, 2016, 8(6), 1084–1098.

[18] A. Wazwaz, *Abundant soliton and periodic wave solutions for the coupled Higgs field equation, the Maccari system and the Hirota–Maccari system*, Physica Scripta, 2012, 85(6), Article ID 065011, 8 pages.

[19] M. Wei, *Bifurcations of traveling wave solutions for a modified Novikov's cubic equation*, Qualitative Theory of Dynamical Systems, 2019, 18(2), 667–686.

[20] Z. Wen, *Bifurcations and exact traveling wave solutions of a new two-component system*, Nonlinear Dynamics, 2017, 87(3), 1917–1922.

[21] Z. Wen, *Bifurcations and exact traveling wave solutions of the celebrated Green-Naghdi equations*, International Journal of Bifurcation and Chaos, 2017, 27(7), Article ID 1750114, 7 pages.

[22] Z. Wen, *The generalized bifurcation method for deriving exact solutions of nonlinear space-time fractional partial differential equations*, Applied Mathematics and Computation, 2020, 366, Article ID 124735, 10 pages.

[23] Z. Wen and K. Zhao, *Qualitative Analysis and Periodic Cusp waves to a Class of Generalized Short Pulse Equations*, Journal of Nonlinear Modeling and Analysis, 2020, 2(4), 565–571.

[24] G. Xu, *New types of doubly periodic standing wave solutions for the coupled Higgs field equation*, Abstract and Applied Analysis, 2014, Article ID 769561, 8 pages.

[25] Q. Zha, *Dynamics of localized wave solutions for the coupled Higgs field equation*, Nonlinear Dynamics, 2020, 101(2), 1181–1198.

[26] Y. Zhou and J. Zhuang, *Bifurcations and Exact Solutions of the Raman Soliton Model in Nanoscale Optical Waveguides with Metamaterials*, Journal of Nonlinear Modeling and Analysis, 2021, 3(1), 145–165.