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Chaos Control and Behavior Analysis of a Discrete-Time Dynamical System with Competitive Effect*

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Abstract This paper is about a class of discrete-time dynamical systems with competitive effects. The local stability of the positive equilibrium point of the system and the conditions for the existence of flip bifurcation and Neimark-Sacker bifurcation are discussed by using the center manifold theorem and bifurcation theory. In addition, the direction of the flip bifurcation and Neimark-Sacker bifurcation is given. Furthermore, a feedback control strategy is employed to control bifurcation and chaos in the system. Finally, flip bifurcation, Neimark-Sacker bifurcation and chaos control strategy are verified with the help of numerical simulations.

Keywords Predator-prey system, flip bifurcation, Neimark-Sacker bifurcation, competitive effect, chaos

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1. Introduction

In nature, no species exists alone, but is closely related to and interdependent with other species. There is not only cooperation but also competition between the various groups. If two populations live together in the same area and compete for limited resources, space and other supplies, both sides of the competition are inhibited. In most cases, only one party benefits while the other party is eliminated, and one party replaces the other. This suggests that two different populations with the same needs cannot live permanently in the same environment. Interspecific competition is one of the most common interactions in predator-prey system. Berryman [1] studied the origin and evolution of predator-prey. Many researchers studied deterministic mathematical models in ecology and the dynamic behavior of prey-predator systems [2–11] . Furthermore, some authors studied the conditions, complexity and stability of spatial pattern formation in prey-predator systems [12–14].

Numerous studies have demonstrated that, for small populations, the discrete-time system is more appropriate than the continuous system. This has been effectively explored and explained in references [15–19]. In addition, Cheng et al. [20] conducted a study on a discrete-time prey-predator system with ratio-dependent

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and Allee effect, and found that the model with logistic growth function had similar bifurcation structures. Recent studies have demonstrated that the discrete-time prey-predator systems have more colorful dynamic behaviors, including bifurcation and chaos, than continuous systems. These dynamic behaviors between populations have been explored and analyzed through numerical simulations (references [21–26]). In [27–30], researchers not only studied the bifurcation phenomenon and dynamic behavior in the prey-predator systems, but also discussed the chaos control strategy for the chaos phenomenon at the unstable equilibrium point. Sarwardi et al. [31] studied a competitive prey-predator system with a prey refuge and obtained the relevant dynamic behaviors.

Interspecific competition is mainly reflected in the competition of different species for resources in the ecosystem (such as food, living space, etc.); interspecific competition is more common among closely related species. In fact, up to now, ecologists still have little understanding of the evolutionary significance of interspecific competition. On the one hand, researchers pay more attention to the form and process of competition, and on the other hand, it takes a long time for evolution to reflect the evolutionary significance of a biological event.

In order to study the effects of competition on populations, in this paper we consider the predator-prey system:

$$\begin{cases} u_{n+1} = u_n + \mu[r_1 u_n(1 - \frac{u_n}{K_1}) - \frac{b u_n v_n}{u_n + v_n} - \varepsilon_1 u_n^2], \\ v_{n+1} = v_n + \mu[r_2 v_n(1 - \frac{v_n}{K_2}) + \frac{a b u_n v_n}{u_n + v_n} - \varepsilon_2 v_n^2], \end{cases} \tag{1.1}$$

where $r_1, r_2, K_1, K_2, \varepsilon_1, \varepsilon_2, a$ and b are greater than zero, r_1 and r_2 are the intrinsic growth rates of the prey u and predator v populations, respectively. b indicates the ability of the predator to consume the prey. ε_1 and ε_2 denote the competition among individuals (i.e., intraspecies interaction) of prey and predator species due to resources. a denotes the conversion rate of the predator to the prey and K_1, K_2 denote environmental carrying capacity of the prey and predator in a particular habitat. And μ expresses the integral step length.

This paper is organized as belows. In Section 2, the existence and stability of the system at different equilibrium points are discussed. In Section 3, we discuss the specific conditions for the existence of Neimark-Sacker bifurcation and flip bifurcation. In Section 4, chaos is controlled to an unstable equilibrium point by the feedback control method. In Section 5, we carry out numerical simulations. Finally, we conclude with a brief summary in the last section.

2. Qualitative study of system

In this section, we will investigate the existence and stability of fixed points in the system. To determine the equilibrium points of equation (1.1), we solve the following set of equations:

$$\begin{cases} u = u + \mu[r_1 u(1 - \frac{u}{K_1}) - \frac{buv}{u+v} - \varepsilon_1 u^2], \\ v = v + \mu[r_2 v(1 - \frac{v}{K_2}) + \frac{abuv}{u+v} - \varepsilon_2 v^2]. \end{cases}$$

By calculation, the following results can be gained directly.

Lemma 2.1. *(i) System* (1.1) *has two positive axial equilibrium points* $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$, $E_2 = (0, \frac{r_2K_2}{r_2+\varepsilon_2K_2})$;

(ii) Assume that system (1.1) *has a unique positive equilibrium point* $E_3 = (u^*, v^*)$, *where* u^* *and* v^* *satisfy*

$$\begin{cases} r_1(1-\frac{u^*}{K_1}) - \frac{bv^*}{u^*+v^*} - \varepsilon_1 u^* = 0, \\ r_2(1-\frac{v^*}{K_2}) + \frac{abu^*}{u^*+v^*} - \varepsilon_2 v^* = 0. \end{cases} \tag{2.1}$$

The Jacobian matrix $H(u,v)$ related to system (1.1) at (u,v) is given by:

$$H(u,v) = \begin{bmatrix} 1+\mu\left(r_1 - \frac{2r_1u}{K_1} - \frac{bv^2}{(u+v)^2} - 2\varepsilon_1 u\right) & -\frac{\mu bu^2}{(u+v)^2} \\ \frac{\mu abv^2}{(u+v)^2} & 1+\mu\left(r_2 - \frac{2r_2v}{K_2} + \frac{abu^2}{(u+v)^2} - 2\varepsilon_2 v\right) \end{bmatrix}.$$

The characteristic equation of $H(u,v)$ can be written as

$$\lambda^2 + m(u,v)\lambda + z(u,v) = 0, \tag{2.2}$$

where

$$\begin{aligned} m(u,v) = &-\mathrm{trH} = -2-\mu\left(r_1 - \frac{2r_1u}{K_1} - \frac{bv^2}{(u+v)^2} - 2\varepsilon_1 u + r_2 - \frac{2r_2v}{K_2} + \frac{abu^2}{(u+v)^2}\right) \\ &+2\mu\varepsilon_2 v, \\ z(u,v) = \det H = &\frac{\mu^2ab^2u^2v^2}{(u+v)^4} + \left[1+\mu\left(r_1 - \frac{2r_1u}{K_1} - \frac{bv^2}{(u+v)^2} - 2\varepsilon_1 u\right)\right]\cdot \\ &\left[1+\mu\left(r_2 - \frac{2r_2v}{K_2} + \frac{abu^2}{(u+v)^2} - 2\varepsilon_2 v\right)\right]. \end{aligned}$$

Suppose that λ_1 and λ_2 are two roots of the characteristic equation of the Jacobian matrix $H|_{(u,v)}$, and we obtain the following definition and results.

Definition 2.1 ([11])**.** The equilibrium point (u,v) is called

(i) Sink if $|\lambda_1| < 1$ and $|\lambda_2| < 1$, and it's locally asymptotically stable;

(ii) Source if $|\lambda_1| > 1$ and $|\lambda_2| > 1$, and it's locally unstable;

(iii) Saddle if either ($|\lambda_1| < 1$ and $|\lambda_2| > 1$) or ($|\lambda_1| > 1$ and $|\lambda_2| < 1$);

(iv) Non-hyperbolic if either $|\lambda_1| = 1$ or $|\lambda_2| = 1$.

The Jacobian matrix for equilibrium point $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$ is:

$$H_{E_1} = \begin{bmatrix} 1-\mu r_1 & -b \\ 0 & 1+\mu(r_2+ab) \end{bmatrix}. \tag{2.3}$$

Then, $\lambda_1 = 1-\mu r_1$ and $\lambda_2 = 1+\mu(r_2+ab)$. Thus, the following propositions hold.

Lemma 2.2. *The eigenvalues at the boundary equilibrium point* $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$ *are* $\lambda_1 = 1-\mu r_1$ *and* $\lambda_2 = 1+\mu(r_2+ab)$, *then*

(i) $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$ *is a saddle, if* $0 < \mu < \frac{2}{r_1}$;

(ii) $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$ *is non-hyperbolic, if* $\mu = \frac{2}{r_1}$;

(iii) $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$ *is a source, if* $\mu > \frac{2}{r_1}$.

Proof. According to (2.3), the two eigenvalues of system (1.1) at the boundary equilibrium point are $\lambda_1 = 1 - \mu r_1$ and $\lambda_2 = 1 + \mu\,(r_2 + ab)$. Since r_1, r_1, a and b are greater than zero, then $|\lambda_2| > 1$. Thus from Definition 2.1, when $|\lambda_1| < 1$, then $0 < \mu < \frac{2}{r_1}$. Thus, $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$ is a saddle. Similarly, when $|\lambda_1| > 1$, then $\mu > \frac{2}{r_1}$, and $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$ is a source. When $\lambda_1 = -1$, then $\mu = \frac{2}{r_1}$, and $E_1 = (\frac{r_1K_1}{r_1+\varepsilon_1K_1}, 0)$ is non-hyperbolic. This completes the proof. □

The Jacobian matrix for $E_2 = (0, \frac{r_2K_2}{r_2+\varepsilon_2K_2})$ is:

$$H_{E_2} = \begin{bmatrix} 1-\mu(b-r_1) & 0 \\ ab & 1-\mu r_2 \end{bmatrix}. \tag{2.4}$$

Then, $\lambda_1 = 1 - \mu(b - r_1), \lambda_2 = 1 - \mu r_2$. Thus, the following results hold.

Lemma 2.3. *The eigenvalues of* H_{E_2} *are* $\lambda_1 = 1 - \mu(b - r_1)$ *and* $\lambda_2 = 1 - \mu r_2$, *then*

(i) $E_2 = (0, \frac{r_2K_2}{r_2+\varepsilon_2K_2})$ *is sink if* $b - r_1 > 0$ *and* $0 < \mu < \min\left\{\frac{2}{r_2}, \frac{2}{b-r_1}\right\}$;

(ii) $E_2 = (0, \frac{r_2K_2}{r_2+\varepsilon_2K_2})$ *is a source if* $b - r_1 > 0$ *and* $\mu > \max\left\{\frac{2}{r_2}, \frac{2}{b-r_1}\right\}$;

(iii) $E_2 = (0, \frac{r_2K_2}{r_2+\varepsilon_2K_2})$ *is non-hyperbolic if* $\mu = \frac{2}{r_2}$ *or* $\mu = \frac{2}{b-r_1}$ *and* $b - r_1 > 0$*;*

(iv) $E_2 = (0, \frac{r_2K_2}{r_2+\varepsilon_2K_2})$ *is a saddle for all possible values of parameters except those values which lies in* (i) *to* (iii)*.*

Proof. (i) According to (2.4), the two eigenvalues of system (1.1) at the boundary equilibrium point E_2 are $\lambda_1 = 1 - \mu(b - r_1)$ and $\lambda_2 = 1 - \mu r_2$. $E_2 = (0, \frac{r_2K_2}{r_2+\varepsilon_2K_2})$ is sink if and only if $|\lambda_1| < 1$ and $|\lambda_2| < 1$. When $|\lambda_1| < 1$, then $0 < \mu < \frac{2}{b-r_1}$, where $b - r_1 > 0$. When $|\lambda_2| < 1$, then $0 < \mu < \frac{2}{r_2}$. In conclusion, $E_2 = (0, \frac{r_2K_2}{r_2+\varepsilon_2K_2})$ is sink if $b - r_1 > 0$ and $0 < \mu < \min\left\{\frac{2}{r_2}, \frac{2}{b-r_1}\right\}$. The same can be proved for (ii), (iii) and (iv). This completes the proof. □

Lemma 2.4 ([11])**.** *Suppose that* $F(\lambda) = \lambda^2 + M\lambda + N$, *and* $F(1) > 0$, λ_1 *and* λ_2 *are roots of* $F(\lambda) = 0$. *Then the following results hold true:*

(i) $|\lambda_1| < 1$ *and* $|\lambda_2| < 1$ *if and only if* $F(-1) > 0$ *and* $N < 1$;

(ii) $|\lambda_1| < 1$ *and* $|\lambda_2| > 1$ *(or* $|\lambda_1| > 1$ *and* $|\lambda_2| < 1$*) if and only if* $F(-1) < 0$;

(iii) $|\lambda_1| > 1$ *and* $|\lambda_2| > 1$ *if and only if* $F(-1) > 0$ *and* $N > 1$;

(iv) $\lambda_1 = -1$ *and* $|\lambda_2| \neq 1$ *if and only if* $F(-1) = 0$ *and* $N \neq 0, 2$;

(v) λ_1 *and* λ_2 *are complex and* $|\lambda_1| = |\lambda_2| = 1$ *if and only if* $M^2 - 4N < 0$ *and* $N = 1$.

By performing calculations, the characteristic equation for system (1.1) can be obtained at the point $E_3(u^*, v^*)$.

$$\lambda^2 + m(u^*, v^*)\lambda + z(u^*, v^*) = 0, \tag{2.5}$$

where

$m(u^*,v^*) = -2 - B\mu,$

$z(u^*,v^*) = A\mu^2 + B\mu + 1,$

$B = r_1 - \frac{2r_1u^*}{K_1} - \frac{bv^{*2}}{(u^*+v^*)^2} - 2\varepsilon_1 u^* + r_2 - \frac{2r_2v^*}{K_2} + \frac{abu^{*2}}{(u^*+v)^2} - 2\varepsilon_2 v^*,$

$A = \left[r_1 - \frac{2r_1u^*}{K_1} - \frac{bv^{*2}}{(u^*+v^*)^2} - 2\varepsilon_1 u^*\right]\left[r_2 - \frac{2r_2v^*}{K_2} + \frac{abu^{*2}}{(u^*+v^*)^2} - 2\varepsilon_2 v^*\right] + \frac{ab^2u^{*2}v^{*2}}{(u^*+v^*)^4}.$

Thus

$$F_{E_3}(\lambda) = \lambda^2 - (2 + B\mu)\lambda + \left(A\mu^2 + B\mu + 1\right).$$

And

$$F(1) = A\mu^2, \quad F(-1) = 4 + 2B\mu + A\mu^2.$$

Using Lemma 2.4, we have the following results:

Lemma 2.5. *System* (1.1) *has the following propositions at* $E_3(u^*,v^*)$.

(i) $E_3(u^*,v^*)$ *is a sink if one of the following conditions is true:*

(1) $B^2 - 4A \geq 0$ *and* $0 < \mu < \frac{-B-\sqrt{B^2-4A}}{A}$,

(2) $B^2 - 4A < 0$ *and* $0 < \mu < \frac{-B}{A}$.

(ii) $E_3(u^*,v^*)$ *is a source if one of the following conditions is true:*

(1) $B^2 - 4A \geq 0$ *and* $\mu > \frac{-B+\sqrt{B^2-4A}}{A}$,

(2) $B^2 - 4A < 0$ *and* $\mu > \frac{-B}{A}$.

(iii) $E_3(u^*,v^*)$ *is non-hyperbolic if one of the following conditions is true:*

(1) $B^2 - 4A \geq 0$ *and* $\mu = \frac{-B\pm\sqrt{B^2-4A}}{A}$,

(2) $B^2 - 4A < 0$ *and* $\mu = \frac{-B}{A}$.

(iv) $E_3(u^*,v^*)$ *is a saddle for all possible values of parameters except those values which lies in* (i) *to* (iii).

Proof. (i) According to Lemma 2.4, $E_3(u^*,v^*)$ is a sink point if and only if $F(1) > 0, F(-1) > 0$ and $N < 1$. It can be obtained by calculation when $B^2 - 4A \geq 0$, then $0 < \mu < \frac{-B-\sqrt{B^2-4A}}{A}$; when $B^2 - 4A < 0$, then $0 < \mu < \frac{-B}{A}$. Therefore, Lemma 2.5 (i) holds. Similarly, Lemma 2.5 (ii), (iii) and (iv) can be established. □

By the above analysis, we can obtain that when the parameters vary on sets $F_{E'_3}$ and $F_{E''_3}$, system (1.1) will have a flip bifurcation at $E_3(u^*,v^*)$, where

$$F_{E'_3} = \left\{(r_1, r_2, a, b, K_1, K_2, \varepsilon_1, \varepsilon_2, \mu) \in \mathbb{R}^9_+ : \mu = \frac{-B - \sqrt{B^2 - 4A}}{A}, B^2 - 4A \geqslant 0\right\},$$

$$F_{E''_3} = \left\{(r_1, r_2, a, b, K_1, K_2, \varepsilon_1, \varepsilon_2, \mu) \in \mathbb{R}^9_+ : \mu = \frac{-B + \sqrt{B^2 - 4A}}{A}, B^2 - 4A \geqslant 0\right\}.$$

When the parameters alter in set F_{E_3}, system (1.1) will have a Neimark-Sacker bifurcation at E_3, where

$$F_{E_3} = \left\{(r_1, r_2, a, b, K_1, K_2, \varepsilon_1, \varepsilon_2, \mu) \in \mathbb{R}^9_+ : \mu = -\frac{B}{A}, B^2 - 4A < 0\right\}.$$

3. Bifurcation phenomenon

3.1. Flip bifurcation

Consider the following system

$$\begin{cases} u_{n+1} = u_n + \mu_1[r_1 u_n(1 - \frac{u_n}{K_1}) - \frac{b u_n v_n}{u_n + v_n} - \varepsilon_1 u_n^2], \\ v_{n+1} = v_n + \mu_1[r_2 v_n(1 - \frac{v_n}{K_2}) + \frac{ab u_n v_n}{u_n + v_n} - \varepsilon_2 v_n^2]. \end{cases} \tag{3.1}$$

The eigenvalues of E_3 are $\lambda_1 = -1$ and $\lambda_2 = 3 + B\mu_1$ with $|\lambda_2| \neq 1$ by Lemma 2.5.

Consider a perturbation corresponding to system (3.1) as follows:

$$\begin{cases} u_{n+1} = u_n + (\mu_1 + \mu^*)[r_1 u_n(1 - \frac{u_n}{K_1}) - \frac{b u_n v_n}{u_n + v_n} - \varepsilon_1 u_n^2], \\ v_{n+1} = v_n + (\mu_1 + \mu^*)[r_2 v_n(1 - \frac{v_n}{K_2}) + \frac{ab u_n v_n}{u_n + v_n} - \varepsilon_2 v_n^2], \end{cases} \tag{3.2}$$

where μ^* is a small perturbation parameter and $|\mu^*| \ll 1$.

Let $p = u - u^*$ and $q = v - v^*$. Then we obtain

$$\begin{pmatrix} p_{n+1} \\ q_{n+1} \end{pmatrix} = \begin{pmatrix} T_{11}p_n + T_{12}q_n + T_{13}p_n^2 + T_{14}p_n q_n + T_{15}q_n^2 + S_{11}\mu^* p_n + S_{12}\mu^* q_n \\ +S_{13}\mu^* p_n^2 + S_{14}\mu^* p_n q_n + S_{15}\mu^* q_n^2 + O((|p_n|, |q_n|, |\mu^*|)^4) \\ T_{21}p_n + T_{22}q_n + T_{23}p_n^2 + T_{24}p_n q_n + T_{25}q_n^2 + S_{21}\mu^* p_n + S_{22}\mu^* q_n \\ +S_{23}\mu^* p_n^2 + S_{24}\mu^* p_n q_n + S_{25}\mu^* q_n^2 + O((|p_n|, |q_n|, |\mu^*|)^4) \end{pmatrix}, \tag{3.3}$$

where

$$T_{11} = 1 + \mu_1[-\frac{r_1}{K_1}u^* + \frac{bu^*v^*}{(u^*+v^*)^2} - \varepsilon_1 u^*], \quad T_{12} = -\frac{\mu_1 b u^{*2}}{(u^*+v^*)^2},$$

$$T_{13} = \mu_1[-\frac{r_1}{K_1} + \frac{bv^{*2}}{(u^*+v^*)^3} - \varepsilon_1], \quad T_{14} = -\frac{2\mu_1 b u^* v^*}{(u^*+v^*)^3}, \quad T_{15} = \frac{\mu_1 b u^*}{(u^*+v^*)^3},$$

$$S_{11} = -\frac{r_1}{K_1}u^* + \frac{bu^*v^*}{(u^*+v^*)^2} - \varepsilon_1 u^*, \quad S_{12} = -\frac{bu^*}{u^*+v^*},$$

$$S_{13} = -\frac{r_1}{K_1} + \frac{bv^{*2}}{(u^*+v^*)^3} - \varepsilon_1, \quad S_{14} = -\frac{2bu^*v^*}{(u^*+v^*)^3}, \quad S_{15} = \frac{bu^{*2}}{(u^*+v^*)^3},$$

$$T_{21} = \frac{\mu_1 ab v^{*2}}{(u^*+v^*)^2}, \quad T_{22} = 1 + \mu_1(-\frac{r_2 v^*}{K_2} - \frac{abu^*v^*}{(u^*+v^*)^2} - \varepsilon_2 v^*),$$

$$T_{23} = -\frac{\mu_1 ab v^{*2}}{(u^*+v^*)^3}, \quad T_{24} = \frac{2\mu_1 abu^*v^*}{(u^*+v^*)^3}, \quad T_{25} = \mu_1(-\frac{r_2}{K_2} - \frac{abu^{*2}}{(u^*+v^*)^3} - \varepsilon_2),$$

$$S_{21} = \frac{abv^{*2}}{(u^*+v^*)^2}, \quad S_{22} = -\frac{r_2 v^*}{K_2} - \frac{abu^*v^*}{(u^*+v^*)^2} - \varepsilon_2 v^*, \quad S_{23} = -\frac{abv^{*2}}{(u^*+v^*)^3},$$

$$S_{24} = \frac{2abu^*v^*}{(u^*+v^*)^3}, \quad S_{25} = -\frac{r_2}{K_2} - \frac{abu^{*2}}{(u^*+v^*)^3} - \varepsilon_2.$$

Construct a nonsingular matrix D_1 and use the following translation:

$$\begin{pmatrix} p \\ q \end{pmatrix} = D_1 \begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix},$$

where

$$D_1 = \begin{pmatrix} T_{12} & T_{12} \\ -1 - T_{11} & \lambda_2 - T_{11} \end{pmatrix}.$$

Taking D_1^{-1} on both sides of system (3.3), we obtain

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix} + \begin{pmatrix} f(p, q, \mu^*) \\ g(p, q, \mu^*) \end{pmatrix}, \tag{3.4}$$

where

$$\begin{aligned}
f(p, q, \mu^*) &= \frac{[T_{13}(\lambda_2 - T_{11}) - T_{12}T_{23}]p^2}{T_{12}(\lambda_2 + 1)} + \frac{[T_{14}(\lambda_2 - T_{11}) - T_{12}T_{24}]pq}{T_{12}(\lambda_2 + 1)} \\
&\quad + \frac{[T_{15}(\lambda_2 - T_{11}) - T_{12}T_{25}]q^2}{T_{12}(\lambda_2 + 1)} + \frac{[S_{11}(\lambda_2 - T_{11}) - T_{12}S_{21}]\mu^* p}{T_{12}(\lambda_2 + 1)} \\
&\quad + \frac{[S_{12}(\lambda_2 - T_{11}) - T_{12}S_{22}]\mu^* q}{T_{12}(\lambda_2 + 1)} + \frac{[S_{13}(\lambda_2 - T_{11}) - T_{12}S_{23}]\mu^* p^2}{T_{12}(\lambda_2 + 1)} \\
&\quad + \frac{[S_{14}(\lambda_2 - T_{11}) - T_{12}S_{24}]\mu^* pq}{T_{12}(\lambda_2 + 1)} + \frac{[S_{15}(\lambda_2 - T_{11}) - T_{12}S_{25}]\mu^* q^2}{T_{12}(\lambda_2 + 1)} \\
&\quad + O((|p|, |q|, |\mu^*|)^4), \\
g(p, q, \mu^*) &= \frac{[T_{13}(1 + T_{11}) + T_{12}T_{23}]p^2}{T_{12}(1 + \lambda_2)} + \frac{[T_{14}(1 + T_{11}) + T_{12}T_{24}]pq}{T_{12}(1 + \lambda_2)} \\
&\quad + \frac{[T_{15}(1 + T_{11}) + T_{12}T_{25}]q^2}{T_{12}(1 + \lambda_2)} + \frac{[S_{11}(1 + T_{11}) + T_{12}S_{21}]\mu^* p}{T_{12}(1 + \lambda_2)} \\
&\quad + \frac{[S_{12}(1 + T_{11}) + T_{12}S_{22}]\mu^* q}{T_{12}(1 + \lambda_2)} + \frac{[S_{13}(1 + T_{11}) + T_{12}S_{23}]\mu^* p^2}{T_{12}(1 + \lambda_2)} \\
&\quad + \frac{[S_{14}(1 + T_{11}) + T_{12}S_{24}]\mu^* pq}{T_{12}(1 + \lambda_2)} + \frac{[S_{15}(1 + T_{11}) + T_{12}S_{25}]\mu^* q^2}{T_{12}(1 + \lambda_2)} \\
&\quad + O((|p|, |q|, |\mu^*|)^4), \\
p &= T_{12}(\tilde{u} + \tilde{v}), \quad q = (\lambda_2 - T_{11})\tilde{v} - (1 + T_{11})\tilde{u}.
\end{aligned}$$

Using the center manifold theorem related to system (3.4) at equilibrium point (0, 0) in a limited region of $\mu^* = 0$. Then there exists a center manifold $W^c(0)$ as follows:

$$W^c(0) = \{(\tilde{u}, \tilde{v}, \mu^*) \in \mathbb{R}^3 : \tilde{v}(\tilde{u}, \mu^*) = T_0\mu^* + T_1\tilde{u}^2 + T_2\tilde{u}\mu^* + T_3\mu^{*2} + O(|\tilde{u}| + |\mu^*|)^3\},$$

and it satisfies

$$H(\tilde{v}(\tilde{u}, \mu^*)) = \tilde{v}(-\tilde{u} + f(p, \tilde{v}(\tilde{u}, \mu^*)) - \lambda_2\tilde{v}(\tilde{u}, \mu^*) - g(p, \tilde{v}(\tilde{u}, \mu^*), \mu^*) = 0,$$

and there exists

$$\begin{aligned}
T_0 &= 0,\\
T_1 &= \frac{[T_{13}(1+T_{11})+T_{12}T_{23}]T_{12}-[T_{14}(1+T_{11})+T_{12}T_{24}](1+T_{11})}{1-\lambda_2^2}\\
&\quad+\frac{[T_{15}(1+T_{11})T_{12}T_{25}](1+T_{11})^2}{(1-\lambda_2^2)T_{12}},\\
T_2 &= \frac{-[S_{11}(1+T_{11})+T_{12}S_{21}]T_{12}+[S_{12}(1+T_{11})+T_{12}S_{22}](1+T_{11})}{T_{12}(1+\lambda_2)^2},\\
T_3 &= 0.
\end{aligned}$$

Therefore, we consider the map restricted to the center manifold $W^c(0)$ as follows:

$$f:\tilde{u}\to -\tilde{u}+t_1\tilde{u}^2+t_2\tilde{u}\mu^*+t_3\tilde{u}^2\mu^*+t_4\tilde{u}\mu^{*2}+t_5\tilde{u}^3+O((|\tilde{u}|+|\mu^*|)^4),$$

where

$$\begin{aligned}
t_1 =&\frac{[T_{13}(\lambda_2-T_{11})-T_{12}T_{23}]T_{12}}{1+\lambda_2}-\frac{[T_{14}(\lambda_2-T_{11})-T_{12}T_{24}](1+T_{11})}{1+\lambda_2}\\
&+\frac{[T_{15}(\lambda_2-T_{11})-T_{12}T_{25}](1+T_{11})^2}{T_{12}(1+\lambda_2)},\\
t_2 =&\frac{[S_{11}(\lambda_2-T_{11})-T_{12}S_{21}]}{1+\lambda_2}-\frac{[S_{12}(\lambda_2-T_{11})-T_{12}S_{22}](1+T_{11})}{T_{12}(1+\lambda_2)},\\
t_3 =&\frac{[T_{13}(\lambda_2-T_{11})-T_{12}T_{23}]2T_2T_{12}}{1+\lambda_2}+\frac{[T_{14}(\lambda_2-T_{11})-T_{12}T_{24}](\lambda_2-2T_{11}-1)T_2}{1+\lambda_2}\\
&-\frac{2[T_{15}(\lambda_2-T_{11})-T_{12}T_{25}](1+T_{11})(\lambda_2-T_{11})T_2}{1+\lambda_2}+\frac{[S_{11}(\lambda_2-T_{11})-T_{12}S_{21}]T_1}{1+\lambda_2}\\
&+\frac{[S_{12}(\lambda_2-T_{11})-T_{12}S_{22}](\lambda_2-T_{11})T_1}{T_{12}(1+\lambda_2)}+\frac{[S_{13}(\lambda_2-T_{11})-T_{12}S_{23}]T_{12}}{1+\lambda_2}\\
&-\frac{[S_{14}(\lambda_2-T_{11})-T_{12}S_{24}](1+T_{11})}{1+\lambda_2}+\frac{[S_{15}(\lambda_2-T_{11})-T_{12}S_{25}](1+T_{11})^2}{(1+\lambda_2)T_{12}},\\
t_4 =&\frac{[S_{11}(\lambda_2-T_{11})-T_{12}S_{21}]T_2}{1+\lambda_2}+\frac{[S_{12}(\lambda_2-T_{11})-T_{12}S_{22}](\lambda_2-T_{11})T_2}{T_{12}(\lambda_2+1)},\\
t_5 =&\frac{[T_{13}(\lambda_2-T_{11})-T_{12}T_{23}]2T_{12}T_1}{1+\lambda_2}+\frac{[T_{14}(\lambda_2-T_{11})-T_{12}T_{24}](\lambda_2-2T_{11}-1)T_1}{\lambda_2+1}\\
&-\frac{2[T_{15}(\lambda_2-T_{11})-T_{12}T_{25}](1+T_{11})(\lambda_2-T_{11})T_1}{(1+\lambda_2)T_{12}}.
\end{aligned}$$

For the flip bifurcation we define the following two nonzero real numbers δ_1 and δ_2, where

$$\delta_1=\left(\frac{\partial^2 f}{\partial\tilde{u}\partial\mu^*}+\frac{1}{2}\frac{\partial f}{\partial\mu^*}\frac{\partial^2 f}{\partial\tilde{u}^2}\right)\bigg|_{(0,0)}=t_2,\quad \delta_2=\left(\frac{1}{6}\frac{\partial^3 f}{\partial\tilde{u}^3}+\left(\frac{1}{2}\frac{\partial^2 f}{\partial\tilde{u}^2}\right)^2\right)\bigg|_{(0,0)}=t_1^2+t_5.$$

From the above analysis, we get the following theorem:

Theorem 3.1. *If $\delta_1 \neq 0, \delta_2 \neq 0$, then system (1.1) passes through a flip bifurcation at the interior equilibrium point E_3 when the parameter μ^* alters in the small region of the point (0,0). In addition, if $\delta_2 > 0$ (resp., $\delta_2 < 0$), then the period-two orbits that bifurcate from equilibrium point E_3 are stable (resp., unstable).*

3.2. Neimark-Sacker bifurcation

Consider a perturbation related to system (1.1) as follows:

$$\begin{cases} u_{n+1} = u_n + (\mu_2 + \mu)[r_1 u_n(1 - \frac{u_n}{K_1}) - \frac{bu_n v_n}{u_n + v_n} - \varepsilon_1 u_n^2], \\ v_{n+1} = v_n + (\mu_2 + \mu)[r_2 v_n(1 - \frac{v_n}{K_2}) + \frac{abu_n v_n}{u_n + v_n} - \varepsilon_2 v_n^2], \end{cases} \tag{3.5}$$

where μ is a limited perturbation parameter and $|\mu| \ll 1$.

The characteristic equation of system (3.5) at E_3 is as follows:

$$\lambda^2 + m(\mu)\lambda + z(\mu) = 0,$$

where

$$m(\mu) = -2 - B(\mu + \mu_2), \quad z(\mu) = A(\mu + \mu_2)^2 + B(\mu + \mu_2) + 1.$$

Since parameters $(r_1, r_2, a, b, K_1, K_2, \varepsilon_1, \varepsilon_2, \mu_2) \in F_{E_3}$, the characteristic values of system (3.5) at the interior equilibrium point E_3 are a pair of complex conjugate numbers λ and $\bar{\lambda}$ as follows.

$$\lambda, \ \bar{\lambda} = \frac{-m(\mu) \pm i\sqrt{4z(\mu) - m^2(\mu)}}{2}.$$

Therefore,

$$\lambda, \ \bar{\lambda} = 1 + \frac{B(\mu_2 + \mu)}{2} \pm \frac{i(\mu_2 + \mu)\sqrt{4A - B^2}}{2},$$

and there exist

$$|\lambda| = |\bar{\lambda}| = z(\mu))^{1/2}, \quad \frac{d|\lambda|}{d\mu}\bigg|_{\mu=0} = \frac{d|\bar{\lambda}|}{d\mu}\bigg|_{\mu=0} = -\frac{B}{2} > 0.$$

When μ changes in a small region of $\mu = 0$, then $\lambda, \bar{\lambda} = c \pm id$, where

$$c = 1 + \frac{\mu_2 B}{2}, \ d = \frac{\mu_2\sqrt{4A - B^2}}{2}.$$

Furthermore, the Neimark-Sacker bifurcation requires that $\mu = 0$, $\lambda^r, \bar{\lambda}^r \neq 1$ (r=1, 2, 3, 4) which is equivalent to $m(0) \neq$ -2, 0, 1, 2. Because parameter $(r_1, r_2, a, b, K_1, K_2, \varepsilon_1, \varepsilon_2, \mu_2) \in F_{E_3}$, consequently $m(0) \neq$ -2, 2. We only require $m(0) \neq$ 0, 1, so that

$$B^2 \neq 2A, \ 3A. \tag{3.6}$$

Let $p = u - u^*$ and $q = v - v^*$. After the transformation of the interior equilibrium point E_3 of system (3.6) to the origin, we have

$$\begin{pmatrix} p_{n+1} \\ q_{n+1} \end{pmatrix} = \begin{pmatrix} T_{11}p_n + T_{12}q_n + T_{13}p_n^2 + T_{14}p_n q_n + T_{15}q_n^2 + O((|p_n|, |q_n|)^3) \\ T_{21}p_n + T_{22}q_n + T_{23}p_n^2 + T_{24}p_n q_n + T_{25}q_n^2 + O((|p_n|, |q_n|)^3) \end{pmatrix}, \tag{3.7}$$

where $T_{ij}(i=1,\ 2,\ 1\leq j\leq 5)$ are given in (3.3) by substituting μ_2 for $\mu_2+\mu$.

Consider the translation as follows:

$$\begin{pmatrix} p \\ q \end{pmatrix} = D_2 \begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix},$$

where

$$D_2 = \begin{pmatrix} T_{12} & 0 \\ c-T_{11} & -d \end{pmatrix}.$$

Taking D_2^{-1} on both sides of system (3.7), we obtain

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix} = \begin{pmatrix} c & -d \\ d & c \end{pmatrix} \begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix} + \begin{pmatrix} \tilde{f}(\tilde{u},\tilde{v}) \\ \tilde{g}(\tilde{u},\tilde{v}) \end{pmatrix},$$

where

$$\begin{aligned}
\tilde{f}(\tilde{u},\tilde{v}) =& \frac{T_{13}p^2}{T_{12}} + \frac{T_{14}pq}{T_{12}} + \frac{T_{15}q^2}{T_{12}} + O((|p|,|q|)^3),\\
\tilde{g}(\tilde{u},\tilde{v}) =& \frac{[T_{13}(c-T_{11})-T_{12}T_{23}]p^2}{T_{12}d} + \frac{[T_{14}(c-T_{11})-T_{12}T_{24}]pq}{T_{12}d}\\
&+ \frac{[T_{15}(c-T_{11})-T_{12}T_{25}]q^2}{T_{12}d} + O((|p|,|q|)^3),\\
p =& T_{12}\tilde{u}, \quad q=(c-T_{11})\tilde{u}-d\tilde{v}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\tilde{f}_{\tilde{u}\tilde{u}} = 2T_{12}T_{13} + 2T_{14}(c-T_{11}) + \frac{2T_{15}(c-T_{11})^2}{T_{12}}, \quad \tilde{f}_{\tilde{u}\tilde{v}} = -T_{14}d - \frac{2T_{15}(c-T_{11})d}{T_{12}},\\
&\tilde{f}_{\tilde{v}\tilde{v}} = \frac{2T_{15}d^2}{T_{12}}, \quad \tilde{f}_{\tilde{u}\tilde{u}\tilde{u}} = \tilde{f}_{\tilde{u}\tilde{u}\tilde{v}} = \tilde{f}_{\tilde{u}\tilde{v}\tilde{v}} = \tilde{f}_{\tilde{v}\tilde{v}\tilde{v}} = 0,\\
&\tilde{g}_{\tilde{u}\tilde{u}} = \frac{2[T_{13}(c-T_{11})-T_{12}T_{23}]T_{12}}{d} + \frac{2[T_{14}(c-T_{11})-T_{12}T_{24}](c-T_{11})}{d}\\
&\qquad + \frac{2[T_{15}(c-T_{11})-T_{12}T_{25}](c-T_{11})^2}{T_{12}d},\\
&\tilde{g}_{\tilde{u}\tilde{v}} = -\frac{[T_{15}(c-T_{11})-T_{12}T_{25}]}{T_{12}}, \quad \tilde{g}_{\tilde{v}\tilde{v}} = -\frac{2[T_{15}(c-T_{11})-T_{12}T_{25}]d}{T_{12}},\\
&\tilde{g}_{\tilde{u}\tilde{u}\tilde{u}} = 0, \quad \tilde{g}_{\tilde{u}\tilde{u}\tilde{v}} = 0, \quad \tilde{g}_{\tilde{u}\tilde{v}\tilde{v}} = 0, \quad \tilde{g}_{\tilde{v}\tilde{v}\tilde{v}} = 0.
\end{aligned}$$

The Neimark-Sacker bifurcation occurs in system (1.1) if the following quantity Λ is not zero

$$\Lambda = -\operatorname{Re}\left[\frac{(1-2\bar{\lambda})\bar{\lambda}^2}{1-\lambda}\Upsilon_{11}\Upsilon_{20}\right] - \frac{1}{2}|\Upsilon_{11}|^2 - |\Upsilon_{02}|^2 + \operatorname{Re}(\bar{\lambda}\Upsilon_{21}), \tag{3.8}$$

where

$$\Upsilon_{11} = \frac{1}{4}\left[(\tilde{f}_{\tilde{u}\tilde{u}} + \tilde{f}_{\tilde{v}\tilde{v}}) + i(\tilde{g}_{\tilde{u}\tilde{u}} + \tilde{g}_{\tilde{v}\tilde{v}})\right],$$

$$\Upsilon_{20} = \frac{1}{8}\left[(\tilde{f}_{\bar{u}\bar{u}} - \tilde{f}_{\bar{v}\bar{v}} + 2\tilde{g}_{\bar{u}\bar{v}}) + i(\tilde{g}_{\bar{u}\bar{u}} - \tilde{g}_{\bar{v}\bar{v}} - 2\tilde{f}_{\bar{u}\bar{v}})\right],$$
$$\Upsilon_{02} = \frac{1}{8}\left[(\tilde{f}_{\bar{u}\bar{u}} - \tilde{f}_{\bar{v}\bar{v}} - 2\tilde{g}_{\bar{u}\bar{v}}) + i(\tilde{g}_{\bar{u}\bar{u}} - \tilde{g}_{\bar{v}\bar{v}} + 2\tilde{f}_{\bar{u}\bar{v}})\right],$$
$$\Upsilon_{21} = \frac{1}{16}\left[(\tilde{f}_{\bar{u}\bar{u}\bar{u}} + \tilde{f}_{\bar{u}\bar{v}\bar{v}} + \tilde{g}_{\bar{u}\bar{u}\bar{v}} + \tilde{g}_{\bar{v}\bar{v}\bar{v}}) + i(\tilde{g}_{\bar{u}\bar{u}\bar{u}} + \tilde{g}_{\bar{u}\bar{v}\bar{v}} - \tilde{f}_{\bar{u}\bar{u}\bar{v}} - \tilde{f}_{\bar{v}\bar{v}\bar{v}})\right].$$

If $\Lambda \neq 0$, Neimark-Sacker bifurcation will occur in system (1.1), and the following theorem holds:

Theorem 3.2. *If the condition* (3.6) *holds,* $\Lambda \neq 0$*, then system* (1.1) *goes through a Neimark-Sacker bifurcation at* E_3 *when the parameter* μ *alters in the small region of the point (0,0). In addition, if* $\Lambda > 0$ *(resp.,* $\Lambda < 0$*), then a repelling (resp., attracting) invariant closed curve bifurcates from equilibrium point* E_3 *for* $\mu < 0$ *(resp.,* $\mu > 0$*).*

4. Chaos control

In this section, we will utilize the feedback control method [27–29] to stabilize the chaotic orbit at an unstable equilibrium point. This will be achieved by adding a feedback control term to system (1.1), which will result in system (1.1) taking the following form:

$$\begin{cases} u_{n+1} = u_n + \mu[r_1 u_n(1 - \frac{u_n}{K_1}) - \frac{b u_n v_n}{u_n + v_n} - \varepsilon_1 u_n^2] - x(u_n, v_n) = f(u_n, v_n), \\ v_{n+1} = v_n + \mu[r_2 v_n(1 - \frac{v_n}{K_2}) + \frac{a b u_n v_n}{u_n + v_n} - \varepsilon_2 v_n^2] = g(u_n, v_n), \end{cases} \tag{4.1}$$

where $x(u_n, v_n) = h_1(u_n - u^*) + h_2(v_n - v^*)$ is the feedback controlling force, h_1 and h_2 are feedback gains, and (u^*, v^*) the unique interior equilibrium point of system (1.1). Furthermore, $f(u^*, v^*) = u^*$, and $g(u^*, v^*) = v^*$.

The Jacobian matrix corresponding to system (4.1) at (u^*, v^*) is as follows:

$$J(u^*, v^*) = \begin{bmatrix} T_{11} - h_1 & T_{12} - h_2 \\ T_{21} & T_{22} \end{bmatrix},$$

where

$$T_{11} = 1 + \mu[-\frac{r_1}{K_1}u^* + \frac{b u^* v^*}{(u^* + v^*)^2} - \varepsilon_1 u^*], \quad T_{12} = -\frac{\mu u^{*2}}{u^* + v^*},$$
$$T_{21} = \frac{\mu a b v^{*2}}{(u^* + v^*)^2}, \quad T_{22} = 1 + \mu(-\frac{r_2 v^*}{K_2} - \frac{a b u^* v^*}{(u^* + v^*)^2} - \varepsilon_2 v^*).$$

Thus, the characteristic equation related to $J(u^*, v^*)$ is:

$$\lambda^2 - (T_{11} + T_{22} - h_1)\lambda + (T_{11} - h_1)T_{22} - (T_{12} - h_2)T_{21} = 0. \tag{4.2}$$

Let λ_1 and λ_2 be the eigenvalues of characteristic equation (4.2). Then

$$\lambda_1 + \lambda_2 = T_{11} + T_{22} - h_1, \quad \lambda_1\lambda_2 = (T_{11} - h_1)T_{22} - (T_{12} - h_2)T_{21}. \tag{4.3}$$

Next, we must solve equations $\lambda_1 = \pm 1$ and $\lambda_1\lambda_2 = 1$ to gain the critical stability line. At the same time, it also ensures that the absolute value λ_1 and λ_2 are less than one.

Suppose that $\lambda_1\lambda_2 = 1$. Then we gain

$$L_1 : T_{11}T_{22} - T_{12}T_{21} - 1 = T_{22}h_1 - T_{21}h_2.$$

Assume that $\lambda_1 = 1$. Then we have

$$L_2 : T_{11} + T_{22} - T_{11}T_{22} + T_{12}T_{21} - 1 = (1 - T_{22})h_1 + T_{21}h_2.$$

Assume that $\lambda_1 = -1$. Then we obtain

$$L_3 : T_{11} + T_{22} + T_{11}T_{22} - T_{12}T_{21} + 1 = (1 + T_{22})h_1 - T_{21}h_2.$$

Thus, the stable eigenvalues lie within the triangular region with the boundaries of the straight lines L_1, L_2, L_3. In addition, when the control parameters h_1 and h_2 take values in the triangular region, system (4.1) will not generate chaos.

5. Numerical simulations

In this section, we draw the bifurcation diagrams, phase portraits, solution of the figures and maximum Lyapunov exponents for system (1.1) to verify the above theoretical analysis.

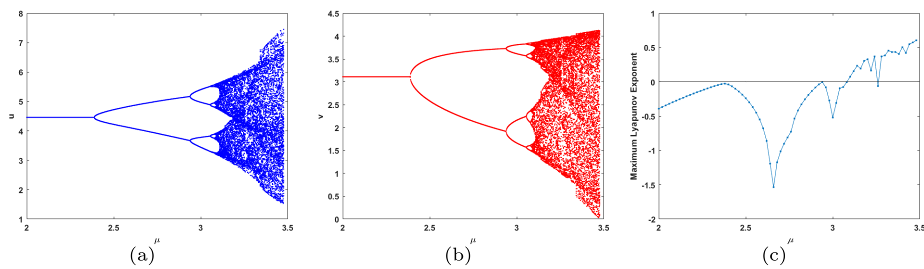


Figure 1. (a,b) Bifurcation diagram corresponding to system (1.1) with $\mu \in [2,\ 3.5]$, $r_1 = 0.8, r_2 = 0.8, a = 0.1, b = 0.5, K_1 = 6, K_2 = 3, \varepsilon_1 = \varepsilon_2 = 0$, and the initial value is (u_0, v_0)=(3, 2). (c) Maximum Lyapunov exponents corresponding to (a,b).

In Figure 1, we consider that the competitive rates of prey and predator $\varepsilon_1 = \varepsilon_2 = 0$ and take μ as the bifurcation parameter to analyze the dynamic behavior of system (1.1) at the interior equilibrium point. Take the parameter values as $(r_1, r_2, a, b, c, K_1, K_2, \varepsilon_1, \varepsilon_2) = (0.8, 0.8, 0.1, 0.5, 6, 3, 0, 0) \in F_{E'_3}$ with the initial value of (u_0, v_0)=(3, 2) and $\mu \in [2,\ 3.5]$. Flip bifurcation appears from the critical value point (4.45905, 3.11045) at $\mu = 2.3880$, and it is stable when $\mu < 2.3880$, and when $\mu > 2.3880$, system (1.1) oscillates with periods of $2, 2^2, 2^3, \cdots$. It can be acquired from Figure 1(c) that chaos will happen in system (1.1) as the bifurcation parameter μ continues to increase.

In Figure 2, we will consider that the competitive rates of prey and predator $\varepsilon_1 = 0.1, \varepsilon_2 = 0$, respectively. Take $(r_1, r_2, a, b, K_1K_2, \varepsilon_1, \varepsilon_2) = (1, 0.8, 0.1, 0.5, 6, 3, 0.1, 0)$ $\in F_{E'_3}$ with the initial value of (u_0, v_0)=(3, 2) and $\mu \in [2.2,\ 3.4]$. Flip bifurcation

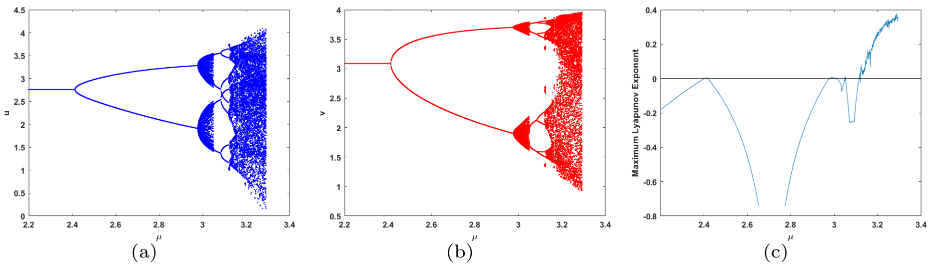


Figure 2. (a,b) Bifurcation diagram corresponding to system (1.1) with $\mu \in [2.2,\ 3.4]$, $r_1 = 1, r_2 = 0.8, a = 0.1, b = 0.5, K_1 = 6, K_2 = 3, \varepsilon_1 = 0.1, \varepsilon_2 = 0$, and the initial value is (u_0, v_0)=(3, 2). (c) Maximum Lyapunov exponents related to (a,b).

appears from the critical value point (2.75981, 3.088) at $\mu = 2.412$, and it is stable when $\mu < 2.412$ and when $\mu > 2.412$, system (1.1) oscillates with periods-two orbits. It can be known from Figure 2(a-c) that the bifurcation at the interior equilibrium point also changes from flip bifurcation to Neimark-Sacker bifurcation and chaos will occur in system (1.1) as the bifurcation parameter μ continues to increase. At the same time, if only the prey is properly competed, it's population density decreases, and the predator population density decreases.

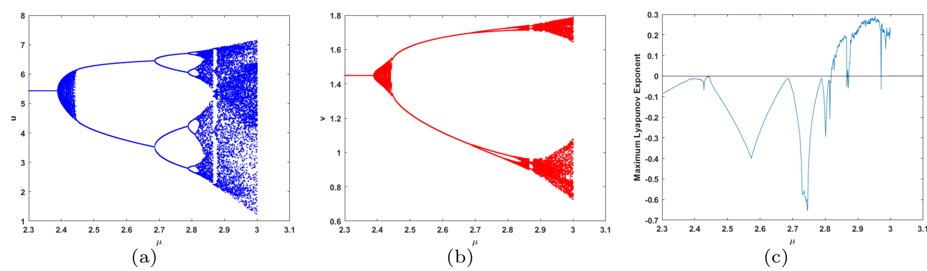
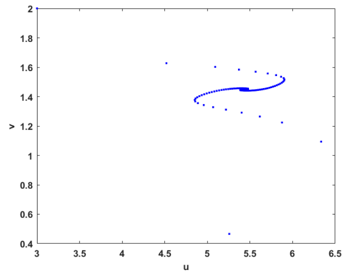


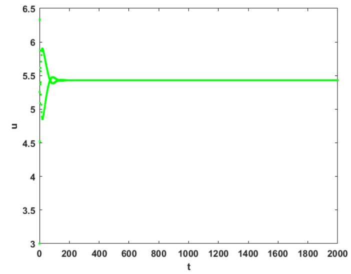
Figure 3. (a,b) Bifurcation diagram of system (1.1) with $\mu \in [2.3,\ 3.1]$, $r_1 = 1, r_2 = 0.8, a = 0.1, b = 0.45, K_1 = 6, K_2 = 3, \varepsilon_1 = 0, \varepsilon_2 = 0.31$, and the initial value is (u_0, v_0)=(3, 2). (c) Maximum Lyapunov exponents related to (a,b).

In Figure 3, we consider that the competitive rates of prey and predator $\varepsilon_1 = 0, \varepsilon_2 = 0.31$, respectively. Take $(r_1, r_2, a, b, K_1, K_2, \varepsilon_1, \varepsilon_2) = (1, 0.8, 0.1, 0.45, 6, 3, 0, 0.31) \in F_{E'_3}$ with the initial value of (u_0, v_0)=(3, 2) and $\mu \in [2.3,\ 3.1]$. Neimark-Sacker bifurcation appears from the critical value point (5.43, 1.45) at $\mu = 2.388$, and it is stable when $\mu < 2.388$ and when $\mu > 2.388$, system (1.1) will change from a Neimark-Sacker bifurcation to a flip bifurcation and finally a chaos phenomenon will occur as the bifurcation parameter μ continues to increase. At the same time, if only the predator is properly competed, it's population density decreases, and the prey population density increases.

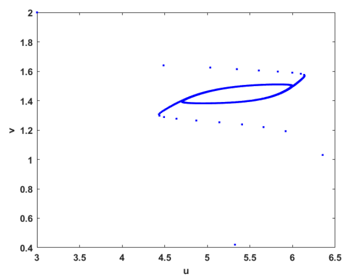
In Figure 5, we consider that the competitive effect of prey and predator $\varepsilon_1 = 0.1, \varepsilon_2 = 0.2$, respectively. Take $(r_1, r_2, a, b, K_1, K_2, \varepsilon_1, \varepsilon_2) = (0.8, 0.8, 0.1, 0.5, 6, 3, 0.1, 0.2) \in F_{E'_3}$ with the initial value of (u_0, v_0)=(3, 2) and $\mu \in [2,\ 3.5]$. Flip bifurcation appears from the critical value point (2.54803, 1.7774) at $\mu = 2.3880$, and it is stable when $\mu < 2.3880$ and when $\mu > 2.3880$, system (1.1) oscillates with



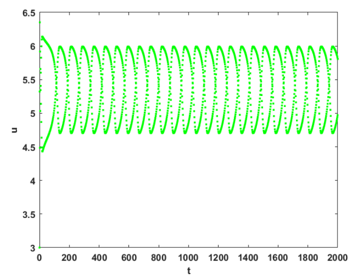
(a) $\mu = 2.35$



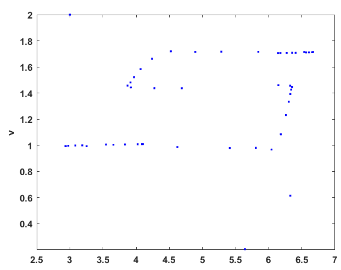
(b) $\mu = 2.35$



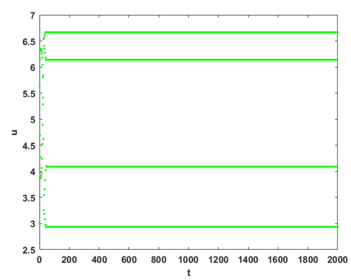
(c) $\mu = 2.42$



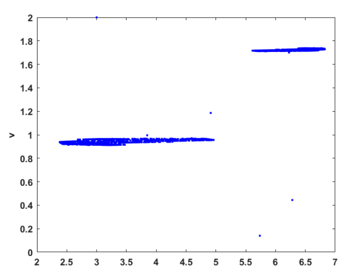
(d) $\mu = 2.42$



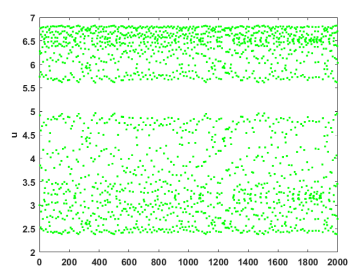
(e) $\mu = 2.75$



(f) $\mu = 2.75$



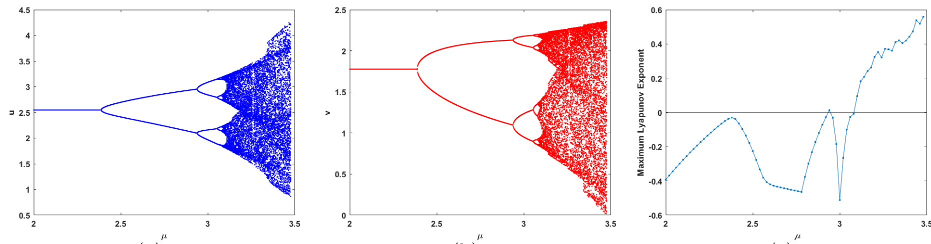
(g) $\mu = 2.85$



(h) $\mu = 2.85$

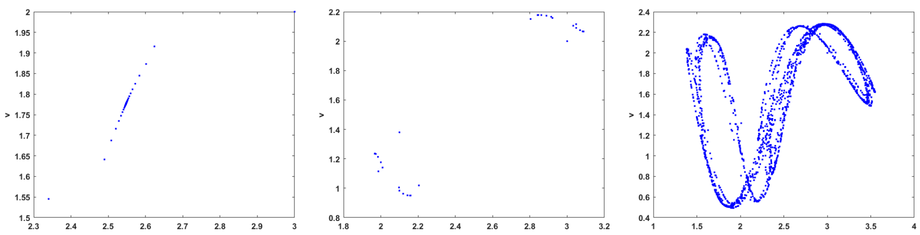
Figure 4. Phase portraits and solution portraits for various values of μ corresponding to Fig. 3.

periods of $2, 2^2, 2^3, \cdots$. It can be known from Figure 6(c) that chaos will occur in system (1.1) as the bifurcation parameter μ continues to increase. At the same time, when $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.2$, system (1.1) will occur not only flip bifurcation and chaos, but also the equilibrium point be lowered, and the prey and the predator population density decreases.

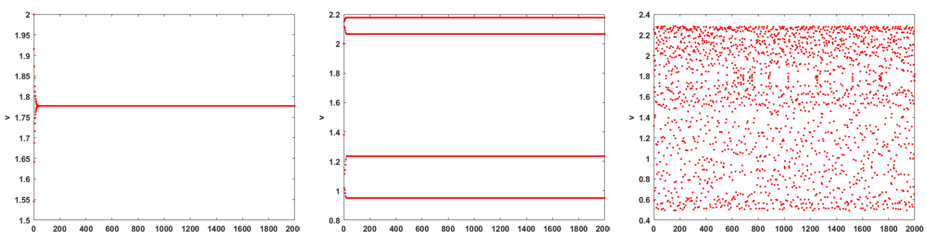


(a) (b) (c)

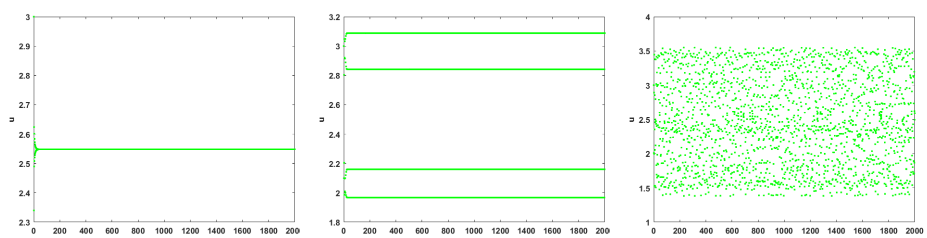
Figure 5. (a,b) Bifurcation diagram corresponding to system (1.1) with $\mu \in [2,\ 3.5]$, $r_1 = 0.8, r_2 = 0.8, a = 0.1, b = 0.5, K_1 = 6, K_2 = 3, \varepsilon_1 = 0.1, \varepsilon_2 = 0.2$, and the initial value is (u_0, v_0)=(3, 2). (c) Maximum Lyapunov exponents related to (a,b).



(a) $\mu = 2.2$ (b) $\mu = 3$ (c) $\mu = 3.3$



(d) $\mu = 2.2$ (e) $\mu = 3$ (f) $\mu = 3.3$

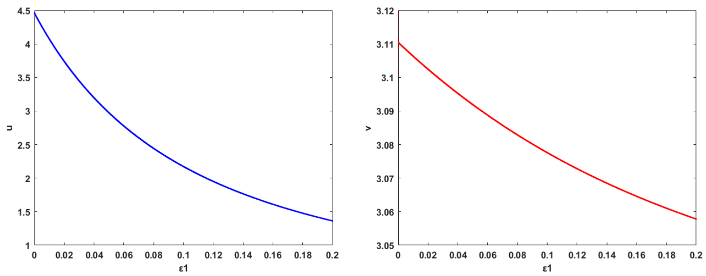


(g) $\mu = 2.2$ (h) $\mu = 3$ (i) $\mu = 3.3$

Figure 6. Phase portraits and solution portraits for various values of μ corresponding to Fig. 5.

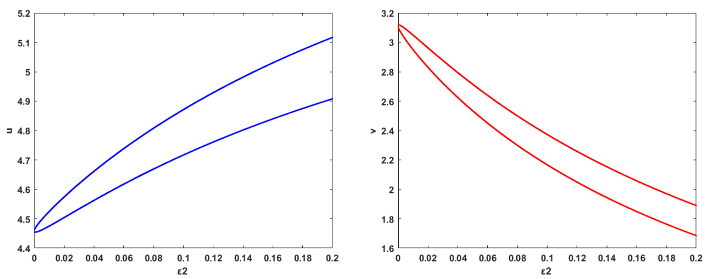
In Figure 7, when the parameter value is $(r_1, r_2, a, b, K_1, K_2, \mu, \varepsilon_2) = (0.8, 0.8, 0.1, 0.5, 6, 3, 2.3880, 0)$ with the initial value of (u_0, v_0)=(3, 2) and $\varepsilon_1 \in [0,\ 0.2]$, ε_1 is bifurcation parameter. At this time, the bifurcation phenomenon of system (1.1) will not occur. The population density of prey and predator will continue to decrease with the increasing of prey competitive effect ε_1.

In Figure 8, when the parameter value is $(r_1, r_2, a, b, K_1, K_2, \mu, \varepsilon_1) = (0.8, 0.8, 0.1,$



(a) Bifurcation diagram for u (b) Bifurcation diagram for v

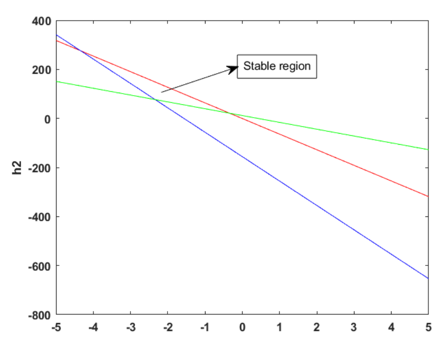
Figure 7. Bifurcation diagram of system (1.1) with $\varepsilon_1 \in [0,\ 0.2]$, $r_1 = 0.8, r_2 = 0.8, a = 0.1, b = 0.5, K_1 = 6, K_2 = 3, \varepsilon_2 = 0, \mu = 2.3880$, and the initial value is (u_0, v_0)=(3, 2).



(a) Bifurcation diagram for u (b) Bifurcation diagram for v

Figure 8. Bifurcation diagram of system (1.1) with $\varepsilon_2 \in [0,\ 0.2]$, $r_1 = 0.8, r_2 = 0.8, a = 0.1, b = 0.5, K_1 = 6, K_2 = 3, \mu = 2.3880, \varepsilon_1 = 0$, and the initial value is (u_0, v_0)=(3, 2).

$0.5, 6, 3, 2.388, 0)$ with the initial value of (u_0, v_0)=(3, 2) and $\varepsilon_2 \in [0,\ 0.2]$, ε_2 is bifurcation parameter. Figure 8 shows the occurrence of period-2 flip bifurcation in system (1.1), the bifurcation graph does not exhibit chaos as the value of parameter ε_2 increases. In addition, the population density of prey and predator will increase and decrease with the increase of predator competitive effect ε_2.



(a)

Figure 9. The bounded region for the eigenvalues of the controlled system (4.1) in the (h_1, h_2) plane.

In Figure 9, take $(r_1, r_2, a, b, K_1, K_2, \varepsilon_1, \varepsilon_2) = (0.8, 0.8, 0.1, 0.5, 6, 3, 0.1, 0.2)$ with the initial value of (u_0, v_0)=(3, 2). In Figure 5(c), when the bifurcation parameter $\mu = 3.3$, system (1.1) will produce chaos. When the h_1 and h_2 are controlled in the triangular region surrounded by three straight lines L_1, L_2, and L_3, the chaos generated by system (4.1) will be controlled near the fixed point and become asymptotically stable state.

6. Conclusions

On the basis of previous study work, this paper studies the stability and bifurcation analysis and chaos control for a class of discrete-time dynamical system with competitive effect. According to the research results, we can have the following results:

(a) System (1.1) has two non-trivial solutions E_1, E_2, in which the stable equilibrium point is positive, reflecting the stable coexistence of prey and predator.

(b) System (1.1) has flip bifurcation and Neimark-Sacker bifurcation at the positive equilibrium point when μ changes in $F_{E'_3}$ or $F_{E''_3}$ and F_{E_3} small fields. It can be seen from Figures 1, 2, 3, 5. We can also find the orbits of periods 2, 4, and 8 periodic windows of flip bifurcation.

(c) When $\varepsilon_1 = 0, \varepsilon_2 \neq 0$, the equilibrium point of system (1.1) changes compared to Figure 1, where u^* goes up and v^* goes down. The number of predators goes down and the number of prey goes up. In addition, the bifurcation phenomenon at the positive equilibrium point also changes from flip bifurcation to Neimark-Sacker bifurcation (see Figures 1, 3).

(d) When $\varepsilon_1 \neq 0, \varepsilon_2 = 0$, the equilibrium point of system (1.1) changes compared to Figure 1, where u^* and v^* both go down. The numbers of predators and prey go down. In addition, the bifurcation phenomenon of system (1.1) at the positive equilibrium point does not change (see Figures 1, 2).

(e) When $\varepsilon_1 \neq 0, \varepsilon_2 \neq 0$, the equilibrium point of system (1.1) changes compared to Figures 1, where u^* and v^* both go down. The density of both predators and prey populations decrease. In addition, the bifurcation phenomenon of system (1.1) at the positive equilibrium point does not change (see Figures 1, 5).

Acknowledgements

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