Journal of Nonlinear Modeling and Analysis https://journal.global-sci.org/jnma
Volume 7, Number 1, February 2025, 91–110 DOI:10.12150/jnma.2025.91

Mathematical Model Dynamics of Cyber Accounts for Vices, Recovery and Relapse

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Abstract In this study, we develop a mathematical model through a system of first-order nonlinear ordinary differential equations. This model covers the dynamics between vulnerable cyber accounts and those implicated in cyber vices such as bullying, scams, spreading of misinformation, and the creation of harmful digital footprints. It further explores the mechanisms of recovery and relapse among these accounts. Through some mathematical analysis, we apply relevant theorems to affirm the model's fundamental properties, which includes its existence, uniqueness, positivity, and boundedness. We also determine the model's cyber vice-free and endemic equilibrium states, analyzing their local and global asymptotic stability based on when the basic reproduction number R_{cb} is greater or less than one. Simulation exercises are conducted to substantiate our theoretical findings and demonstrate the model's behavior in relation to R_{cb}. The simulation outcomes reveal an escalating trend in cyber vices, showing the necessity for targeted interventions that promote a more secure online environment for users and the broader cyber space.

Keywords Local stability, global stability, positivity and boundedness, basic reproduction number R_{cb}

MSC(2010) 92B05, 92B20.

1. Introduction

Cyber vices, including cyberbullying, scams, fraud, fake news, and digital footprints, have emerged as formidable challenges in the digital era. Understanding their dynamics and developing effective strategies to combat them is crucial for ensuring online safety and security. The advent of the digital age has revolutionized the way we communicate, work, and interact with each other [23]. The internet and social media platforms have brought numerous opportunities for connectivity and information sharing. However, along with these advancements, there has also been a rise in online cyber vices, posing significant challenges for individuals, organizations, and societies as a whole [22]. Cyberbullying, scams, fraud, fake news, and the creation of digital footprints are just a few examples of the detrimental activities that can occur in the online realm [19–21]. Understanding the dynamics of online cyber vices and developing effective strategies to combat them is of utmost importance in today's interconnected world. Mathematical modeling provides a powerful tool for gaining understanding into the mechanisms driving these vices and can aid

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in the development of proactive measures to mitigate their impact. By constructing mathematical models, interactions between different entities involved in cyber vices and recovery dynamics can be studied [11–15]. Additionally, mathematical models enable us to study the effectiveness of various intervention strategies and assess their potential outcomes before implementation. In recent years, there has been a significant research focus on studying the effects of cyber defense and attacks using mathematical models, with notable contributions from Alexopoulous and Daras [1], Emmanuello and Ridley [2], Gencoglu [3], and Guilan, Del-Rey, and Cassado [4]. Furthermore, several researchers [9, 10] and [16–18], have developed mathematical models to address security threats and issues in the cyber space. Additionally, the cyber analysis of smart power and grid processes has been explored through mathematical modeling by researchers such as [5–8]. Despite these advancements, to the best of our understanding, we have identified a crucial gap regarding the understanding of how cyber accounts/sites are created and utilized to perpetrate vices such as scams, frauds, bullying, and leaving digital footprints. Moreover, the potential mitigation of these cyber activities using mathematical modeling has yet to be comprehensively discussed. To address this gap, our study seeks to investigate and analyze cyber accounts and their role in fostering various vices. We propose a robust mathematical model to shed light on this important aspect of cyber behavior. In Section 2, we provide a detailed formulation of the mathematical model, aiming to capture the cyber activities. Subsequently, in Section 3, we present the mathematical analysis of the existence and uniqueness, positivity, boundedness, and stability properties of the model. To gain understanding and verify the model's performance, we perform numerical simulations in Section 4. Finally, in Section 5, we conclude our study, summarizing the key findings and proposing potential future directions for the work.

2. Mathematical model formulation

The model assumes a deterministic system, where the future behavior of the system is entirely determined by its initial conditions and parameter values. This assumption disregards any random or stochastic elements that may exist in the real-world dynamics of cyber vices and recovery. The following system of equations describes the dynamics of various online vices, including cyber bullying, scams, fake news, and digital footprints, as well as the recovery process of affected online accounts. The total number of online cyber accounts ($N_{cb}(t)$) is divided into compartments such that $N_{cb}(t) = V_a(t) + C_b(t) + S_c(t) + F_n(t) + D_f(t) + R_s(t)$, where the rate of change of vulnerable cyber accounts (V_a) is given by

$$\frac{dV_a}{dt} = \Phi - (\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f)V_a - \delta V_a. \tag{2.1}$$

The rate of change of cyber account dedicated to cyber bullying (C_b) is

$$\frac{dC_b}{dt} = \beta_1 C_b V_a - (\delta + \psi_1)C_b + \varsigma_1 C_b. \tag{2.2}$$

The rate of change of cyber account involved in scams (S_c) is

$$\frac{dS_c}{dt} = \beta_2 S_c V_a - (\delta + \psi_2 S_c + \varsigma_2 C_b. \tag{2.3}$$

The rate of change of cyber account spreading fake news (F_n) is

$$\frac{dF_n}{dt} = \beta_3 F_n V_a - (\delta + \psi_3)F_n + \varsigma_3 C_b. \tag{2.4}$$

The rate of change of cyber account leaving digital footprints (D_f) is

$$\frac{dD_f}{dt} = \beta_4 D_f V_a - (\delta + \psi_4)D_f + \varsigma_4 C_b. \tag{2.5}$$

The rate of change of cyber account (R_s) is

$$\frac{dR_s}{dt} = \psi_1 C_b + \psi_2 S_c + \psi_3 F_n + \psi_4 D_f - \delta R_s - \varsigma_1 C_b - \varsigma_2 S_c - \varsigma_3 F_n - \varsigma_4 D_f. \tag{2.6}$$

Coupling the equations (2.1) - (2.6) above yields

$$\left.\begin{aligned}
&\frac{dV_a}{dt} = \Phi - (\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f)V_a - \delta V_a,\\
&\frac{dC_b}{dt} = \beta_1 C_b V_a - (\delta + \psi_1)C_b + \varsigma_1 C_b,\\
&\frac{dS_c}{dt} = \beta_2 S_c V_a - (\delta + \psi_2 S_c + \varsigma_2 S_c,\\
&\frac{dF_n}{dt} = \beta_3 F_n V_a - (\delta + \psi_3)F_n + \varsigma_3 F_n,\\
&\frac{dD_f}{dt} = \beta_4 D_f V_a - (\delta + \psi_4)D_f + \varsigma_4 D_f,\\
&\frac{dR_s}{dt} = \psi_1 C_b + \psi_2 S_c + \psi_3 F_n + \psi_4 D_f - \delta R_s,\\
&- \varsigma_1 C_b - \varsigma_2 S_c - \varsigma_3 F_n - \varsigma_4 D_f.
\end{aligned}\right\} \tag{2.7}$$

Subject to initial conditions $V_a \geq 0, C_b \geq 0, S_c \geq 0, F_n \geq 0, D_f \geq 0, R_s \geq 0$. The parameters in (2.7) are defined as follows:

- Φ: Creation of new cyber sites and platforms.
- δ: Inactivation/death rate of cyber accounts.
- $\beta_i (i = 1-4)$: Contact rates between vulnerable account and accounts devoted to cyber vices.
- $\psi_i (i = 1 - 4)$: Progression rates affected cyber accounts to recovery.
- $\varsigma_i (i = 1 - 4)$: Relapse rate of the accounts for cyber vices after recovery.

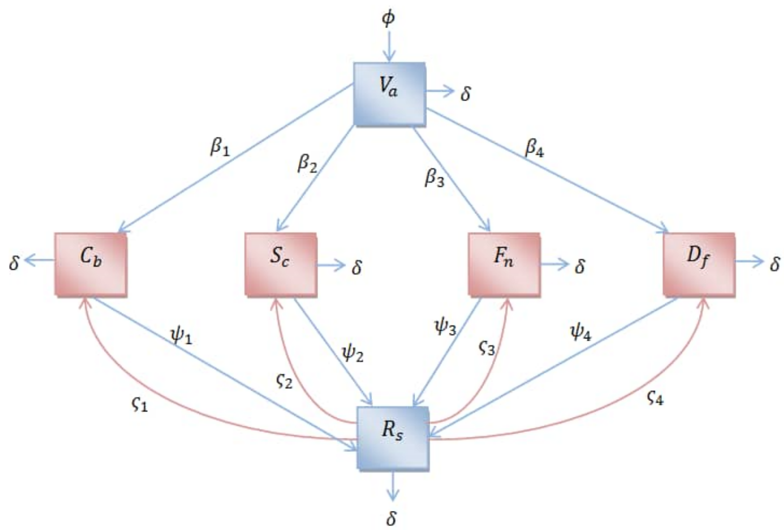

Figure 1. Schematic diagram for the cyber accounts interactions in the host cyber space

3. Basic analysis

The first order ordinary differential equation (ODE) in general form is

$$\dot{\varphi} = \hbar(t, \varpi), \ \varphi(t_o) = \varphi_o. \tag{3.1}$$

Theorem 3.1. *Let ∇ denote the domain such that*

$$|t - t_o| \leq x, \ |\varphi - \varphi_o| \leq y, \ \varphi = (\varphi_1, \varphi_2, ..., \varphi_n). \tag{3.2}$$

Assume that

$$\hbar(t, \varphi) : ||\hbar(t, \varphi_1) - \hbar(t, \varphi_2)|| \leq q^* ||\varphi_1 - \varphi_2|| \tag{3.3}$$

satisfies the Lipschitz condition and the pairs (t, φ_1) and (t, φ_2) are in ∇, where q^* denotes a positive constant, then there exists $\delta > 0$ for the interval $|t - t_o| \leq \delta$, and a unique continuous vector solution $\varphi(t)$ of (3.1) exists such that (3.1) is satisfied by $\frac{\partial \hbar_k}{\partial W_l}$, $k, l = 1, 2, ...n$ in domain ∇ is continuous and bounded.

Lemma 3.1. *If the continuous partial derivative $\hbar(t, \varphi)$, that is $\frac{\partial \hbar_k}{\partial W_l}$, exists for a bounded close convex domain $\mathbb{R}$, then for $\mathbb{R}$ the Lipschitz condition is satisfied such that $0 < \mathbb{R} < \infty$.*

Proof. From (2.7), we assume that

$$\left.\begin{aligned}
&W_1 = \Phi - (\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f)V_a - \delta V_a,\\
&W_2 = \beta_1 C_b V_a - (\delta + \psi_1)C_b + \varsigma_1 C_b,\\
&W_3 = \beta_2 S_c V_a - (\delta + \psi_2 S_c + \varsigma_1 C_b,\\
&W_4 = \beta_3 F_n V_a - (\delta + \psi_3)F_n + \varsigma_1 C_b,\\
&W_5 = \beta_4 D_f V_a - (\delta + \psi_4)D_f + \varsigma_1 C_b,\\
&W_6 = \psi_1 C_b + \psi_2 S_c + \psi_3 F_n + \psi_4 D_f - \delta R_s,\\
&- \varsigma_1 C_b - \varsigma_2 S_c - \varsigma_3 F_n - \varsigma_4 D_f.
\end{aligned}\right\} \tag{3.4}$$

Then from the first expression in (3.4), we obtain

$$\left.\begin{aligned}
\frac{\partial W_1}{\partial V_a} &= |-(\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f) - \delta| < \infty,\\
\frac{\partial W_1}{\partial C_b} &= |-\beta_1 V_a| < \infty,\\
\frac{\partial W_2}{\partial S_c} &= |-\beta_2 V_a| < \infty,\\
\frac{\partial W_3}{\partial F_n} &= |-\beta_3 V_a| < \infty,\\
\frac{\partial W_4}{\partial D_f} &= |-\beta_4 V_a| < \infty,\\
\frac{\partial W_5}{\partial R_s} &= |0| < \infty.
\end{aligned}\right\} \tag{3.5}$$

Following the same approach for the remaining variables W_b, W_c, W_d, W_e and W_f in (3.4), we observe that the partial derivatives of the remaining variables exist, and are continuous and bounded in the domain. Hence, a unique solution exists. □

Theorem 3.2. *The solutions of model (2.7) under its initial conditions are positive for time $t > 0$.*

Proof. Taking the first expression in (2.7), that is

$$\begin{aligned}
\frac{dV_a}{dt} &= Q - (\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f)V_a - \delta V_a\\
&\geq -(\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f)V_a - \delta V_a,
\end{aligned} \tag{3.6}$$

then

$$\int \frac{dV_a}{V_a} \geq -\int ((\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f) - \delta)dt, \tag{3.7}$$

where

$$\ln V_a \geq -((\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f) - \delta)t + C. \tag{3.8}$$

In (3.8), C denotes a constant and at $t = 0, \ln V_a(0) = C$. On substitution,

$$\ln V_a(t) \geq -((\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f) - \delta)t + \ln V_a(0), \tag{3.9}$$

where

$$\ln V_a(t) - \ln V_a(0) \geq -((\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f) - \delta)t. \tag{3.10}$$

Therefore,

$$\ln \frac{V_a(t)}{V_a(0)} \geq -((\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f) - \delta)t \tag{3.11}$$

and

$$V_a(t) \geq V_a(0)e^{-((\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f) - \delta)t} > 0. \tag{3.12}$$

The same approach applies to the remaining state variables of (2.7), ensuring the positivity of the model. □

Theorem 3.3. *The set $\mathbb{R}^{+6}$ is invariant in the positive sense with respect to (2.7), such that the solution of (2.7) is uniformly bounded in the subset*

$$\Upsilon = \left\{ (V_a, C_b, S_c, F_n, D_f, R_s) \in \mathbb{R}^{+6} : N_{cb} \leq \frac{\phi}{\mu} \right\}.$$

Proof. The total host cyber accounts yield

$$\left.\begin{aligned} \frac{dN_{cb}}{dt} &= \frac{dN_{cb}}{dt} + \frac{dN_{cb}}{dt} + \frac{dN_{cb}}{dt} + \frac{dN_{cb}}{dt} + \frac{dN_{cb}}{dt} \\ &= \phi + \delta(V_a + C_b + S_c + F_n + D_f + R_s) \leq \phi - \delta N_{cb}. \end{aligned}\right\} \tag{3.13}$$

The solution to the differential inequality (3.13) yields

$$\lim_{t\to\infty} \sup N_{cb}(t) \leq \frac{\phi}{\delta}. \tag{3.14}$$

It follows from (3.14) that model (2.7) is uniformly bounded in Υ, that is $\Upsilon = \left\{ (V_a, C_b, S_c, F_n, D_f, R_s) \in \mathbb{R}^{+6} : N_{cb} \leq \frac{\phi}{\mu} \right\}$. This shows that (2.7) is well behaved and meaningful in the sense of transmission of vices in the cyber host space. □

The equilibrium solutions are determined to study the long term behavior of the model. The two equilibria are the cyber vices free (CB^1)and endemic (CB^2) equilibrium solutions which are given by

$$CB^1 = \left(\frac{\Phi}{\delta}, 0, 0, 0, 0, 0 \right), \tag{3.15}$$

and

$$\left.\begin{aligned} CB^2 = V_a^* &= \frac{(\delta + \psi_1 + \varsigma_1)}{\beta_1}, \\ C_b^* &= \frac{\delta(\Phi\beta_1 - \delta - \psi_1 - \varsigma_1)}{\beta_1(\delta + \psi_1 + \varsigma_1)}, \\ S_c^* &= \frac{\delta(\Phi\beta_2 - \delta - \psi_2 - \varsigma_2)}{\beta_2(\delta + \psi_2 + \varsigma_2)}, \\ F_n^* &= \frac{\delta(\Phi\beta_3 - \delta - \psi_3 - \varsigma_3)}{\beta_3(\delta + \psi_3 + \varsigma_3)}, \\ D_f^* &= \frac{\delta(\Phi\beta_4 - \delta - \psi_4 - \varsigma_4)}{\beta_4(\delta + \psi_4 + \varsigma_4)}, \\ R_n^* &= \frac{\Phi(\beta_1 - \delta - \psi_1 - \delta\varsigma_1)(\psi_1 - \varsigma_1)}{\beta_1(\delta + \psi_1 + \varsigma_1)\delta} \\ &\times \frac{\Phi(\beta_2 - \delta - \psi_2 - \delta\varsigma_2)(\psi_2 - \varsigma_2)}{\beta_2(\delta + \psi_2 + \varsigma_2)\delta} \\ &\times \frac{\Phi(\beta_3 - \delta - \psi_3 - \delta\varsigma_3)(\psi_3 - \varsigma_3)}{\beta_3(\delta + \psi_3 + \varsigma_3)\delta} \\ &\times \frac{\Phi(\beta_4 - \delta - \psi_4 - \delta\varsigma_4)(\psi_4 - \varsigma_4)}{\beta_4(\delta + \psi_4 + \varsigma_4)\delta}. \end{aligned}\right\} \tag{3.16}$$

Using the equilibrium solution (CB^1) in (3.15), we establish the following theorem.

Theorem 3.4. *The basic reproductive number R_{cb} is given by*

$$\rho(FV^{-1}) = R_{cb} = \frac{\Phi(\beta_1 + \beta_2 + \beta_3 + \beta_4)}{(\delta(\delta + \psi_1 + \varsigma_1))(\delta(\delta + \psi_2 + \varsigma_2))(\delta(\delta + \psi_3 + \varsigma_3))(\delta(\delta + \psi_4 + \varsigma_4))}.$$

Proof. Using the next generation matrix method [18], the non-negative matrix $\mathbb{F}$, known as the appearance of new cyber vices in the model yields

$$\mathbb{F} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta_1 \Phi}{\delta} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_2 \Phi}{\delta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_3 \Phi}{\delta} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_4 \Phi}{\delta} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3.17}$$

while the matrix $\mathbb{V}$ known as the transitions of cyber account in and out of the system is given by

$$\mathbb{V} = \begin{pmatrix} \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta + \psi_1 + \varsigma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta + \psi_2 + \varsigma_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta + \psi_3 + \varsigma_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta + \psi_4 + \varsigma & 0 \\ 0 & \psi_1 - \varsigma_1 & \psi 2 - \varsigma_2 & \psi_3 - \varsigma_3 & \psi_4 - \varsigma_4 & \delta \end{pmatrix}, \tag{3.18}$$

while the inverse of $\mathbb{V}$ yields

$$\mathbb{V}^{-1} = \begin{pmatrix} \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta + \psi_1 + \varsigma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta + \psi_2 + \varsigma_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta + \psi_3 + \varsigma_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta + \psi_4 + \varsigma_4 & 0 \\ 0 & \psi_1 - \varsigma_1 & \psi_2 - \varsigma_2 & \psi_3 - \varsigma_3 & \psi_4 - \varsigma_4 & \delta \end{pmatrix}, \tag{3.19}$$

so that $\mathbb{F}\mathbb{V}^{-1}$ yields

$$\mathbb{F}\mathbb{V}^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta_1\,\Phi}{\delta\,(\delta+\psi_1+\varsigma_1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_2\,\Phi}{\delta\,(\delta+\psi-2+\varsigma_2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_3\,\Phi}{\delta\,(\delta+\psi_3+\varsigma-3)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_4\,\Phi}{\delta\,(\delta+\psi_4+\varsigma_4)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.20}$$

The eigenvalues of $\mathbb{F}\mathbb{V}^{-1}$ become

$$\mathbb{F}\mathbb{V}^{-1} = \begin{pmatrix} 0 \\ 0 \\ \frac{\beta_4\,\Phi}{\delta\,(\delta+\psi_4+\varsigma_4)} \\ \frac{\beta_3\,\Phi}{\delta\,(\delta+\psi_3+\varsigma_3)} \\ \frac{\beta_2\,Q}{\delta\,(\delta+\psi_2+\varsigma_2)} \\ \frac{\beta_1\,\Phi}{\delta\,(\delta+\psi_1+\varsigma_1)} \end{pmatrix}. \tag{3.21}$$

Therefore the basic reproductive number is the spectral radius

$$\rho(\mathbb{F}\mathbb{V}^{-1}) = R_{cb} = \frac{\Phi(\beta_1+\beta_2+\beta_3+\beta_4)}{(\delta(\delta+\psi_1+\varsigma_1))(\delta(\delta+\psi_2+\varsigma_2))(\delta(\delta+\psi_3+\varsigma_3))(\delta(\delta+\psi_4+\varsigma_4))}. \tag{3.22}$$

This is the average number of secondary cases of cyber accounts with vices generated when a affected cyber account is introduced into the vulnerable cyber space.

Theorem 3.5. *The cyber vices free equilibrium solution CB^1 (3.15) is locally asymptotically stable if $R_{cb} < 1$ and unstable otherwise.*

Proof. We obtain the Jacobian $\mathbb{J}$ of model (2.7) at the cyber vices free equilibrium solution (3.15),

$$\mathbb{J} = \begin{pmatrix} \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta_1\Phi}{\delta} - (\delta+\psi_1+\varsigma_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_2\Phi}{\delta} - (\delta+\psi_2+\varsigma_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_3\Phi}{\delta} - (\delta+\psi_3+\varsigma_3) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_4\Phi}{\delta} - (\delta+\psi_4+\varsigma) & 0 \\ 0 & \psi_1-\varsigma_1 & \psi_2-\varsigma_2 & \psi_3-\varsigma_4 & \psi_4-\varsigma_4 & \delta \end{pmatrix}. \tag{3.23}$$

It is observed that only two eigenvalues (δ) are negative, while the remaining positive eigenvalues are $\frac{\Phi(\beta_1+\beta_2+\beta_3+\beta_4)}{(\delta(\delta+\psi_1+\varsigma_1))(\delta(\delta+\psi_2+\varsigma_2))(\delta(\delta+\psi_3+\varsigma_3))(\delta(\delta+\psi_4+\varsigma_4))}$, which form R_{cb}. Therefore, when $1 - R_{cb} > 0$, then $-R_{cb} > -1$, such that $R_{cb} < 1$. This shows that the cyber vices free equilibrium solution is locally asymptotically stable. □

Theorem 3.6. *The cyber vices free equilibrium is globally asymptotically stable when $R_{cb} < 1$.*

Proof. By the use of the comparison technique, the cyber accounts dedicated to online vices alone can be simplified as

$$\begin{pmatrix} \dot{C}_b \\ \dot{S}_c \\ \dot{F}_n \\ \dot{D}_f \end{pmatrix} = \left(\frac{V_a}{N_{cb}}\right) \mathbb{F} \begin{pmatrix} C_b \\ S_c \\ F_n \\ D_f \end{pmatrix} - \mathbb{V} \begin{pmatrix} C_b \\ S_c \\ F_n \\ D_f \end{pmatrix} = \mathbb{F} - \mathbb{V} \begin{pmatrix} C_b \\ S_c \\ F_n \\ D_f \end{pmatrix} - \left(1 - \frac{V_a}{N_{cb}}\right)\mathbb{F} \begin{pmatrix} C_b \\ S_c \\ F_n \\ D_f \end{pmatrix} \leq$$

$$(\mathbb{F} - \mathbb{V}) \begin{pmatrix} C_b \\ S_c \\ F_n \\ D_f \end{pmatrix}, \tag{3.24}$$

where $\mathbb{F}$ and $\mathbb{V}$ are defined in (3.17) and (3.18). The linearized differential inequality in (3.24) is stable whenever $R_{cb} < 1$. Consequently, by standard comparison, $(C_b, S_c, F_n, D_f) \to (0, 0, 0, 0)$ as $t \to \infty$. If one substitutes $C_b = S_c = F_n = D_f = 0$ in (2.7), then $(V_a, R) \to \left(\frac{\Phi}{\delta}, 0\right)$ as $t \to \infty$. Therefore, $(V_a, C_b, S_c, F_n, D_f, R) \to \left(\frac{\Phi}{\delta}, 0, 0, 0, 0, 0\right)$ as $t \to \infty$ and hence the CB^1 is globally asymptotically stable when $R_{cb} < 1$. This implies that cyber vices can be eliminated from the cyber space if R_{cb} can be maintained below 1. □

Theorem 3.7. *The cyber vices endemic equilibrium (3.16) is locally asymptotically stable whenever $R_{cb} > 1$.*

Proof. Linearizing the model using the cyber-vices endemic equilibrium solution (3.16), we obtain the Jacobian $\mathbb{J}$ as

$$\mathbb{J} = \begin{pmatrix} -A_o & c_o & c_1 & c_2 & c_3 & 0 \\ b_o & A_1 & 0 & 0 & 0 & 0 \\ b_1 & 0 & A_2 & 0 & 0 & 0 \\ b_2 & 0 & 0 & A_3 & 0 & 0 \\ b_3 & 0 & 0 & 0 & A_4 & 0 \\ 0 & \psi_1 - \varsigma_1 & \psi_2 - \varsigma_2 & \psi_3 - \varsigma_3 & \psi_4 - \varsigma_4 & \delta \end{pmatrix}, \tag{3.25}$$

where the following are defined:

$$\left.\begin{array}{l} A_o = -C_b\beta_1 - D_f\beta_4 - F_n\beta_3 - S_c\beta_2 - \delta,\ A_1 = V_a\beta_1 - \delta - \psi_1 - \varsigma_1, \\ A_2 = Va\beta_2 - \delta - \psi_2 - \varsigma_2\ \ A_3 = Va\beta_3 - \delta - \psi_4 - \varsigma_3, \\ A_4 = Va\beta_4 - \delta - \psi_4 - \varsigma_4\ \ c_o = \beta_1 V_a\ \ c_1 = \beta_2 V_a\ \ c_2 = \beta_3 V_a, \\ c_3 = \beta_4 V_a\ \ c_4 = \beta_5 V_a,\ b_o = \beta_1 C_b,\ b_1 = \beta_2 S_c,\ b_2 = \beta_3 F_n,\ \beta_4 D_f. \end{array}\right\} \tag{3.26}$$

The characteristics equation yields

$$\lambda^6 + \tau_o\lambda^5 + \tau_1\lambda^4 + \tau_2\lambda^3 + \tau_3\lambda^2 + \tau_4\lambda + \tau_5, \tag{3.27}$$

where

$$\tau_o = \left(-\delta - A_4 - A_3 - A_2 - A_1 + A_o\right), \tag{3.28}$$

$$\left.\begin{array}{l} \tau_1 = \Big(A_2\, A_1 + A_3\, A_1 + A_4\, A_1 - A_1\, A_o + \delta\, A_1 + A_3\, A_2 + A_4\, A_2 - \\ \quad A_2\, A_o + \delta\, A_2 + A_4\, A_3 - A_3\, A_o + \delta\, A_3 - A_4\, A_o + \delta\, A_4 - \delta \\ \quad A_o - b_1\, c_1 - b_2\, c_2 - b_3\, c_3 - b_o\, c_o\Big), \end{array}\right\} \tag{3.29}$$

$$\left.\begin{array}{l} \tau_2 = \Big(- A_1\, A_2\, A_3 - A_1\, A_2\, A_4 + A_1\, A_2\, A_o - A_1\, A_2\, \delta - A_1\, A_3\, A_4 + A_1\, A_3\, A_o - \\ \quad A_1\, A_3\, \delta + A_1\, A_4\, A_o - A_1\, A_4\, \delta + A_1\, A_o\, \delta + b_1\, c_1\, A_1 + b_2\, c_2\, A_1 + b_3\, c_3\, A_1 \\ \quad - A_2\, A_3\, A_4 + A_2\, A_3\, A_o - A_2\, A_3\, \delta + A_2\, A_4\, A_o - A_2\, A_4\, \delta + A_2\, A_o\, \delta + b_2\, c_2 \\ \quad A_2 + b_3\, c_3\, A_2 + A_2\, b_o\, c_o + A_3\, A_4\, A_o - A_3\, A_4\, \delta + A_3\, A_o\, \delta + A_3\, b_1\, c_1 + b_3\, c_3 \\ \quad A_3 + A_3\, b_o\, c_o + A_4\, A_o\, \delta + A_4\, b_1\, c_1 + A_4\, b_2\, c_2 + A_4\, b_o\, c_o + b_1\, c_1\, \delta + b_2\, c_2 \\ \quad \delta + b_3\, c_3\, \delta + b_o\, c_o\, \delta\Big), \end{array}\right\} \tag{3.30}$$

$$\left.\begin{array}{l} \tau_3 = \Big(A_1\, A_2\, A_3\, A_4 - A_1\, A_2\, A_3\, A_o + A_1\, A_2\, A_3\, \delta - A_1\, A_2\, A_4\, A_o + A_1\, A_2\, A_4 \\ \quad \delta - A_1\, A_2\, A_o\, \delta - A_1\, A_2\, b_2\, c_2 - A_1\, A_2\, b_3\, c_3 - A_1\, A_3\, A_4\, A_o + A_1\, A_3\, A_4 \\ \quad \delta - A_1\, A_3\, A_o\, \delta - A_1\, A_3\, b_1\, c_1 - A_1\, A_3\, b_3\, c_3 - A_1\, A_4\, A_o\, \delta - A_1\, A_4\, b_1\, c_1 \\ \quad - A_1\, A_4\, b_2\, c_2 - A_1\, b_1\, c_1\, \delta - A_1\, b_2\, c_2\, \delta - A_1\, b_3\, c_3\, \delta - A_2\, A_3\, A_4\, A_o + A_2 \\ \quad A_3\, A_4\, \delta - A_2\, A_3\, A_o\, \delta - A_2\, A_3\, b_3\, c_3 - A_2\, A_3\, b_o\, c_o - A_2\, A_4\, A_o\, \delta - A_2\, A_4 \\ \quad b_2\, c_2 - A_2\, A_4\, b_o\, c_o - A_2\, b_2\, c_2\, \delta - A_2\, b_3\, c_3\, \delta - A_2\, b_o\, c_o\, \delta - A_3\, A_4\, A_o \\ \quad \delta - A_3\, A_4\, b_1\, c_1 - A_3\, A_4\, b_o\, c_o - A_3\, b_1\, c_1\, \delta - A_3\, b_3\, c_3\, \delta - A_3\, b_o\, c_o\, \delta - \\ \quad A_4\, b_1\, c_1\, \delta - A_4\, b_2\, c_2\, \delta - A_4\, b_o\, c_o\, \delta\Big), \end{array}\right\} \tag{3.31}$$

$$\left.\begin{aligned}\tau_4 =&\Big(A_1\,A_2\,A_3\,A_4\,A_o - A_1\,A_2\,A_3\,A_4\,\delta + A_1\,A_2\,A_3\,A_o\,\delta + A_1\,A_2\,A_3\,b_3\,c_3\\ &+ A_1\,A_2\,A_4\,A_o\,\delta + A_1\,A_2\,A_4\,b_2\,c_2 + A_1\,A_2\,b_2\,c_2\,\delta + A_1\,A_2\,b_3\,c_3\,\delta +\\ &A_1\,A_3\,A_4\,A_o\,\delta + A_1\,A_3\,A_4\,b_1\,c_1 + A_1\,A_3\,b_1\,c_1\,\delta + A_1\,A_3\,b_3\,c_3\,\delta +\\ &A_1\,A_4\,b_1\,c_1\,\delta + A_1\,A_4\,b_2\,c_2\,\delta + A_2\,A_3\,A_4\,A_o\,\delta + A_2\,A_3\,A_4\,b_o\,c_o +\\ &A_2\,A_3\,b_3\,c_3\,\delta + A_2\,A_3\,b_o\,c_o\,\delta + A_2\,A_4\,b_2\,c_2\,\delta + A_2\,A_4\,b_o\,c_o\,\delta + A_3\\ &A_4\,b_1\,c_1\,\delta + A_3\,A_4\,b_o\,c_o\,\delta\Big),\end{aligned}\right\} \tag{3.32}$$

and

$$\left.\begin{aligned}\tau_5 =& -\delta\,\Big(A_1\,A_2\,A_3\,A_4\,A_o + A_1\,A_2\,A_3\,b_3\,c_3 + A_1\,A_2\,A_4\,b_2\,c_2 + A_1\,A_3\,A_4\\ &b_1\,c_1 + A_2\,A_3\,A_4\,b_o\,c_o\Big).\end{aligned}\right\} \tag{3.33}$$

From (3.28)-(3.33) it can be shown that $\tau_i > 0$ for $i = 0 - 5$ using the following

$$\left.\begin{aligned}&\mathbb{D}_1 = \tau_o, \mathbb{D}_2 = \begin{pmatrix}\tau_o & 1\\ \tau_2 & \tau_1\end{pmatrix}, \mathbb{D}_3 = \begin{pmatrix}\tau_o & 1 & 0\\ \tau_2 & \tau_1 & \tau_o\\ 0 & 0 & \tau_2\end{pmatrix}, \mathbb{D}_4 = \begin{pmatrix}\tau_o & 1 & 0 & 0\\ \tau_2 & \tau_1 & \tau_o & 0\\ 0 & \tau_3 & \tau_2 & \tau_1\\ 0 & 0 & 0 & \tau_3\end{pmatrix},\\ &\mathbb{D}_5 = \begin{pmatrix}\tau_o & 1 & 0 & 0 & 0\\ \tau_2 & \tau_1 & \tau_o & 1 & 0\\ \tau_4 & \tau_3 & \tau_2 & \tau_1 & \tau_o\\ 0 & 0 & \tau_4 & \tau_3 & \tau_2\\ 0 & 0 & 0 & 0 & \tau_4\end{pmatrix}, \mathbb{D}_6 = \begin{pmatrix}\tau_o & 1 & 0 & 0 & 0 & 0\\ \tau_2 & \tau_1 & \tau_o & 1 & 0 & 0\\ \tau_4 & \tau_3 & \tau_2 & \tau_1 & \tau_o & 1\\ 0 & \tau_5 & \tau_4 & \tau_3 & \tau_2 & \tau_1\\ 0 & 0 & 0 & \tau_5 & \tau_4 & \tau_3\\ 0 & 0 & 0 & 0 & 0 & \tau_5\end{pmatrix}.\end{aligned}\right\} \tag{3.34}$$

In (3.34), if all $\tau_i > 0$ for $i = 0 - 5$ and the conditions in $\mathbb{D}_1 - \mathbb{D}_6$ hold, then the cyber vices endemic equilibrium is locally asymptotically stable when $R_{cb} > 1$. □

Theorem 3.8. *The cyber vices present equilibrium is globally asymptotically stable if $R_{cb} > 1$.*

Proof. At equilibrium, model (2.7) becomes

$$\left.\begin{aligned}&\phi = (\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f)V_a - \delta V_a,\\ &(\delta + \psi_1 + \varsigma_1) = \beta_1 V_a,\\ &(\delta + \psi_2 + \varsigma_1) = \beta_2 V_a,\\ &(\delta + \psi_3 + \varsigma_1) = \beta_3 V_a,\\ &(\delta + \psi_4 + \varsigma_1) = \beta_4 V_a,\\ &R_s = \psi_1 C_b + \psi_2 S_c + \psi_3 F_n + \psi_4 D_f - \delta R_s,\\ &-\varsigma_1 C_b - \varsigma_2 S_c - \varsigma_3 F_n - \varsigma_4 D_f.\end{aligned}\right\} \tag{3.35}$$

We construct a Goh-Volterra Lyapunov L_{cb} function given by

$$\left.\begin{aligned} L_{cb} =&\Big(V_a - V_a^* - V_a^* \ln \frac{V_a}{V_a^*}\Big) + \Big(C_b - C_b^* - C_b^* \ln \frac{C_b}{C_b^*}\Big) \\ &+\Big(S_c - S_c^* - S_c^* \ln \frac{S_c}{S_c^*}\Big) + \Big(F_n - F_n^* - F_n^* \ln \frac{F_n}{F_n^*}\Big) \\ &+\Big(D_f - D_f^* - D_f^* \ln \frac{D_f}{D_f^*}\Big). \end{aligned}\right\} \tag{3.36}$$

Differentiating (3.36), we have

$$\left.\begin{aligned} \dot{L_{cb}} =&\Big(1 - \frac{V_a^*}{V_a}\Big)\dot{V_a} + \Big(1 - \frac{C_b^*}{C_b}\Big)\dot{C_b} \\ &+\Big(1 - \frac{S_c^*}{S_c}\Big)\dot{S_c} + \Big(1 - \frac{F_n^*}{F_n}\Big)\dot{F_n} \\ &+\Big(1 - \frac{D_f^*}{D_f}\Big)\dot{D_f}. \end{aligned}\right\} \tag{3.37}$$

In (3.37), each term is solved directly. For the first term, we obtain

$$\left.\begin{aligned} \dot{L_{cb}} =& \Big(1 - \frac{V_a^*}{V_a}\Big)((\beta_1 C_b + \beta_2 S_c + \beta_3 F_n + \beta_4 D_f)V_a - \delta V_a \\ &- (\beta_1 C_b^* - \beta_2 S_c^* - \beta_3 F_n^* - \beta_4 D_f^*)V_a^* + \delta V_a^*)) \\ \dot{L_{cb}} =& \Big(1 - \frac{V_a^*}{V_a}\Big)(\beta_1(C_b V_a - C_b^* V_a^*) + \beta_2(S_c V_a - S_c^* V_a^*) \\ &+ \beta_3(F_n V_a - F_n^* V_a^*) + \beta_4(D_f V_a - D_f^* V_a^*) - \delta(V_a - V_a^*) \\ \dot{L_{cb}} =& \beta_1\Big(1 - \frac{C_b^* V_a^*}{C_b V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a C_b}{V_a V_a^* C_b^*}\Big) \\ &+ \beta_2\Big(1 - \frac{S_c^* V_a^*}{S_c V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a S_c}{V_a V_a^* S_c^*}\Big) \\ &+ \beta_3\Big(1 - \frac{F_n^* V_a^*}{F_n V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a F_n}{V_a V_a^* F_n^*}\Big) \\ &- \delta\Big(2 - \frac{V_a^*}{V_a} - \frac{V_a}{V_a^*}\Big). \end{aligned}\right\} \tag{3.38}$$

For the second term we have

$$\dot{L_{cb}} = \beta_1\Big(1 - \frac{C_b^*}{C_b}\Big)(V_a - V_a^*) = \beta_1\Big(1 - \frac{C_b^* V_a^*}{C_b V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a C_b}{V_a V_a^* C_b^*}\Big). \tag{3.39}$$

For the third term, we get

$$\dot{L_{cb}} = \beta_2\Big(1 - \frac{S_c^*}{S_c}\Big)(V_a - V_a^*) = \beta_2\Big(1 - \frac{S_c^* V_a^*}{S_c V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a S_c}{V_a V_a^* S_c^*}\Big). \tag{3.40}$$

For the fourth term, we obtain

$$\dot{L_{cb}} = \beta_3\Big(1 - \frac{F_n^*}{F_n}\Big)(V_a - V_a^*) = \beta_3\Big(1 - \frac{F_n^* V_a^*}{F_n V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a F_n}{V_a V_a^* F_n^*}\Big). \tag{3.41}$$

And the fifth term yields

$$\dot{L}_{cb} = \beta_4\Big(1 - \frac{D_f^*}{D_f}\Big)(V_a - V_a^*) = \beta_4\Big(1 - \frac{D_f^* V_a^*}{D_f V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a D_f}{V_a V_a^* D_f^*}\Big). \tag{3.42}$$

Finally, the sixth term becomes

$$\left.\begin{aligned} &(\psi_1 - \varsigma_1)\Big(1 - \frac{R_s^* C_b^*}{R_s C_b} - \frac{C_b^*}{C_b} + \frac{R_s^* R_s C_b}{R_s R_s^* C_b^*}\Big) \\ &+ (\psi_2 - \varsigma_2)\Big(1 - \frac{R_s^* S_c^*}{R_s S_c} - \frac{S_c^*}{S_c} + \frac{R_s^* R_s S_c}{R_s R_s^* S_c^*}\Big) + \\ &(\psi_3 - \varsigma_3)\Big(1 - \frac{R_s^* F_n^*}{R_s F_n} - \frac{F_n^*}{F_n} + \frac{R_s^* R_s F_n}{R_s R_s^* F_n^*}\Big) \\ &+ (\psi_4 - \varsigma_4)\Big(1 - \frac{R_s^* D_f^*}{R_s D_f} - \frac{D_f^*}{D_f} + \frac{R_s^* R_s D_f}{R_s R_s^* D_f^*}\Big) - \\ &\delta\Big(2 - \frac{R_s^*}{R_s} - \frac{R_s}{R_s^*}\Big). \end{aligned}\right\} \tag{3.43}$$

Coupling (3.39) - (3.43), we obtain

$$\left.\begin{aligned} &\Big(1 - \frac{V_a^*}{V_a}\Big)(\beta_1(C_b V_a - C_b^* V_a^*) + \beta_2(S_c V_a - S_c^* V_a^*) + \beta_3(F_n V_a - F_n^* V_a^*) \\ &+ \beta_4(D_f V_a - D_f^* V_a^*) - \delta(V_a - V_a^*)\beta_1\Big(1 - \frac{C_b^* V_a^*}{C_b V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a C_b}{V_a V_a^* C_b^*}\Big) + \\ &\beta_2\Big(1 - \frac{S_c^* V_a^*}{S_c V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a S_c}{V_a V_a^* S_c^*}\Big) + \beta_3\Big(1 - \frac{F_n^* V_a^*}{F_n V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a F_n}{V_a V_a^* F_n^*}\Big) \\ &- \delta\Big(2 - \frac{V_a^*}{V_a} - \frac{V_a}{V_a^*}\Big) + \beta_1\Big(1 - \frac{C_b^* V_a^*}{C_b V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a C_b}{V_a V_a^* C_b^*}\Big) + \\ &\beta_2\Big(1 - \frac{S_c^* V_a^*}{S_c V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a S_c}{V_a V_a^* S_c^*}\Big) + \beta_3\Big(1 - \frac{F_n^* V_a^*}{F_n V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a F_n}{V_a V_a^* F_n^*}\Big) + \\ &\beta_4\Big(1 - \frac{D_f^* V_a^*}{D_f V_a} - \frac{V_a^*}{V_a} + \frac{V_a^* V_a D_f}{V_a V_a^* D_f^*}\Big) + (\psi_1 - \varsigma_1)\Big(1 - \frac{R_s^* C_b^*}{R_s C_b} - \frac{C_b^*}{C_b} + \frac{R_s^* R_s C_b}{R_s R_s^* C_b^*}\Big) \\ &+ (\psi_2 - \varsigma_2)\Big(1 - \frac{R_s^* S_c^*}{R_s S_c} - \frac{S_c^*}{S_c} + \frac{R_s^* R_s S_c}{R_s R_s^* S_c^*}\Big) + (\psi_3 - \varsigma_3)\Big(1 - \frac{R_s^* F_n^*}{R_s F_n} - \\ &\frac{F_n^*}{F_n} + \frac{R_s^* R_s F_n}{R_s R_s^* F_n^*}\Big) + (\psi_4 - \varsigma_4)\Big(1 - \frac{R_s^* D_f^*}{R_s D_f} - \frac{D_f^*}{D_f} + \frac{R_s^* R_s D_f}{R_s R_s^* D_f^*}\Big) - \delta\Big(2 - \frac{R_s^*}{R_s} - \frac{R_s}{R_s^*}\Big). \end{aligned}\right\} \tag{3.44}$$

In (3.34), if $V_a = V_a^*, C_b = C_b^*, S_c = S_c^*, F_n = F_n^*, D_f = D_f^*$ and $R_s = R_s^*$, then $\dot{L}_{cb} \geq 0 \Leftrightarrow (V_a, C_b, S_c, F_n, D_f, R_n) = (V_a^*, C_b^*, S_c^*, F_n^*, D_f^*, R_n^*)$. Hence from the Lasalle invariance princple [18], the cyber vices endemic equilibrium is globally asymptotically stable when $R_{cb} > 1$. □

4. Numerical simulations and discussions

Here, we perform numerical simulations using the ODE 45 in python computational software and the following initial values were assumed in millions $V_b = 10, C_b =$

$5, S_c = 3, F_n = 2, D_f = 1, R_c = 0.9$ while the parameter values are given in Table 1, as seen in existing literature.

Table 1. Parameter Definitions

Descriptions	Parameters	Range Values	Point Values per day	Sources
Recruitment rate of new cyber accounts	Φ	0-1	0.8	[1–3, 12–14]
Inactivation/death rate of accounts	δ	0-1	0.8354	[1–3, 12–14]
Interaction/contact rates	$\beta_i(i = 1 - 4)$	0-1	0.21, 0.34, 0.41, 0.50	[1–3, 12–14]
Recovery rates	$\psi_i(i = 1 - 4)$	0-1	0.11, 0.21, 0.31, 0.42	[1–3, 12–14]
Relapse rates	$\varsigma_i(i = 1 - 4)$	0-1	0.11, 0.55, 0.77, 0.8	[1–3, 12–14]

In Figures 2-5, we observe the convergence of the model's stability behavior towards the cyber vices free equilibrium solution when $R_{cb} < 1$, varied within different days. This implies that without any intervention or control measures, the cyber vices, including attacks, bullying, scams, and digital footprints, continue to increase over time in the cyberspace. In the context of the simulations, the convergence of the model's stability behavior to the vices-free equilibrium when $R_{cb} < 1$ refers to the long-term behavior of the system when it is not influenced by any external factors, interventions, or control measures. At the vices-free equilibrium, the cyber vices, such as attacks, bullying, scams, and digital footprints, are effectively minimized, resulting in a stable and safe cyberspace environment. In this scenario, the model indicates that over time, the system will settle into a state where the prevalence of cyber vices remains low, and there is little to no impact on the cyberspace. Similarly, Figures 6-9 illustrate the convergence of the model's stability behavior towards the cyber vices endemic equilibrium solution. The sustained increase in cyber vices over time emphasizes the urgent need for effective measures to curb their propagation and minimize their impact on the cyberspace. On the other hand, the convergence of the model's stability behavior to the vices endemic equilibrium, when $R_{cb} > 1$ refers to the long-term behavior of the system where the presence of cyber vices becomes a persistent feature in the cyberspace. In this scenario, the model shows that without intervention, the prevalence of cyber vices will continue to grow, potentially leading to a detrimental impact on the cyberspace and its users.The simulations illustrate that in the absence of proactive measures to address cyber vices, the system tends to converge towards the vices endemic equilibrium. This highlights the critical importance of implementing strategies to curb the propagation of cyber vices and reduce their impact. Without such measures, the cyberspace becomes increasingly vulnerable to malicious activities and harmful practices.

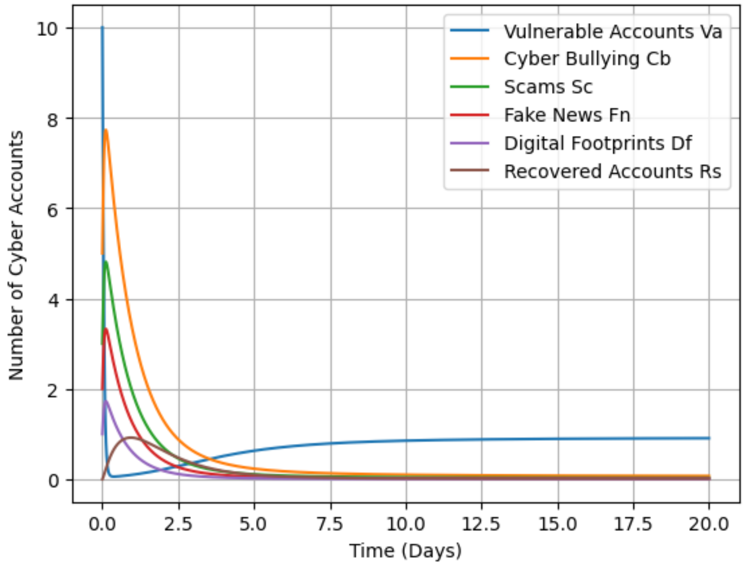

Figure 2. The behavior of the total cyber accounts model variables varying within 20 days at the cyber vices free equilibrium when $R_{cb} < 1$.

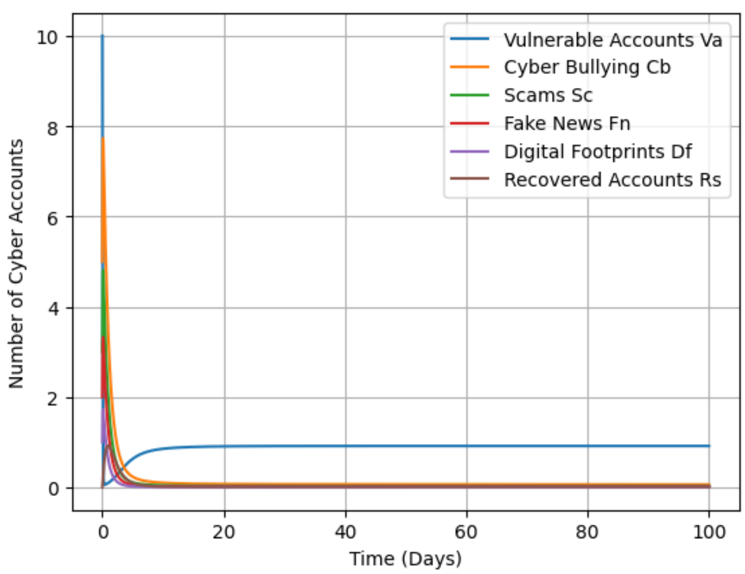

Figure 3. The behavior of the total cyber accounts model variables varied in 100 days at the cyber vices free equilibrium when $R_{cb} < 1$.

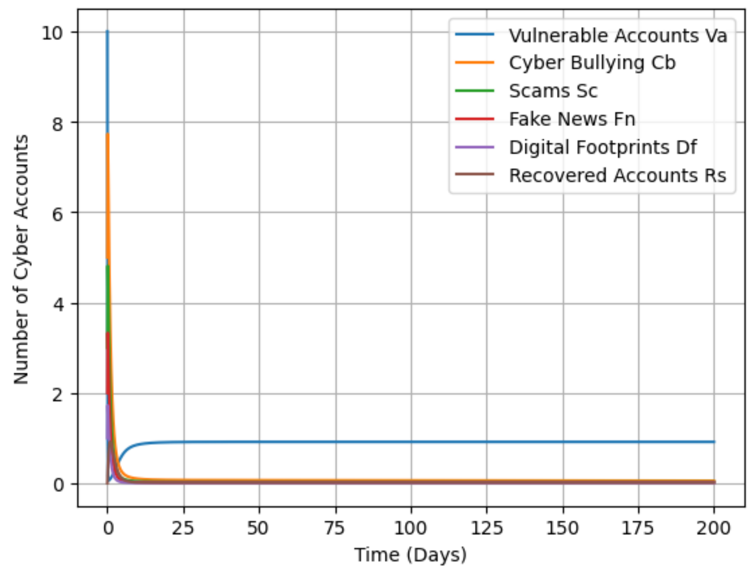

Figure 4. The behavior of the total cyber accounts model variables varied in 200 days at the cyber vices free equilibrium when $R_{cb} < 1$.

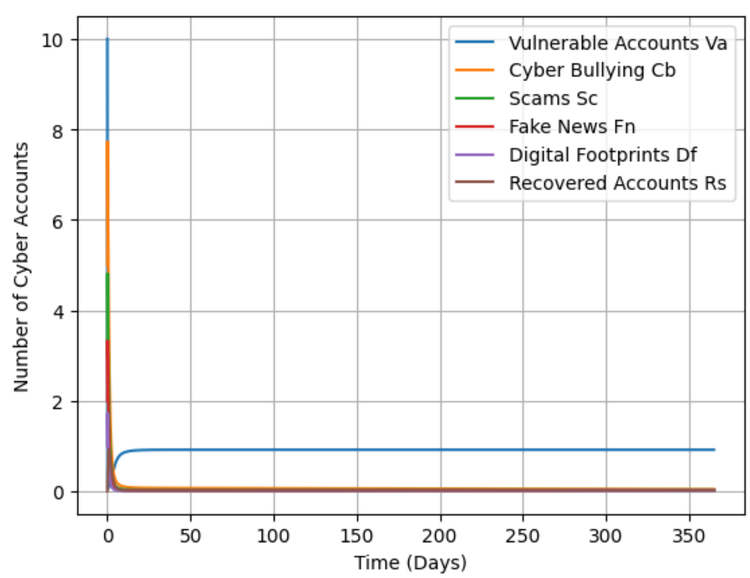

Figure 5. The behavior of the total cyber accounts model variables varied within 365 days at the cyber vices free equilibrium when $R_{cb} < 1$.

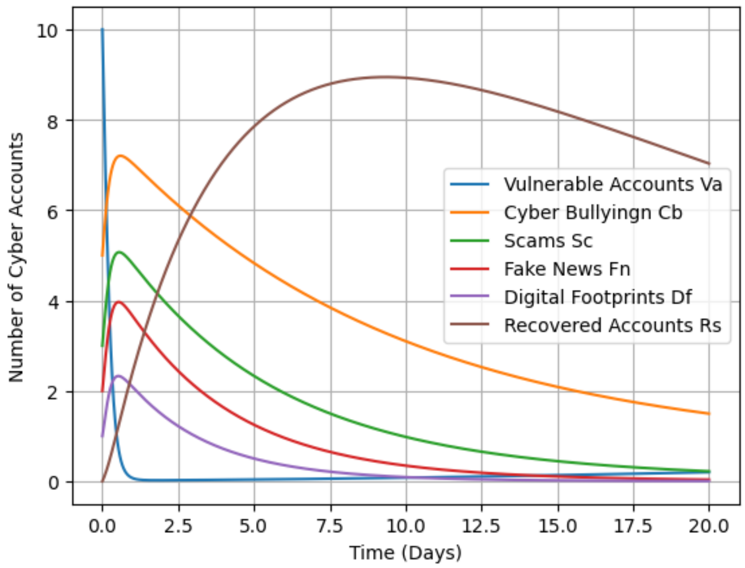

Figure 6. The behavior of the total cyber accounts model variables varied in 20 days at the cyber vices free equilibrium when $R_{cb} > 1$.

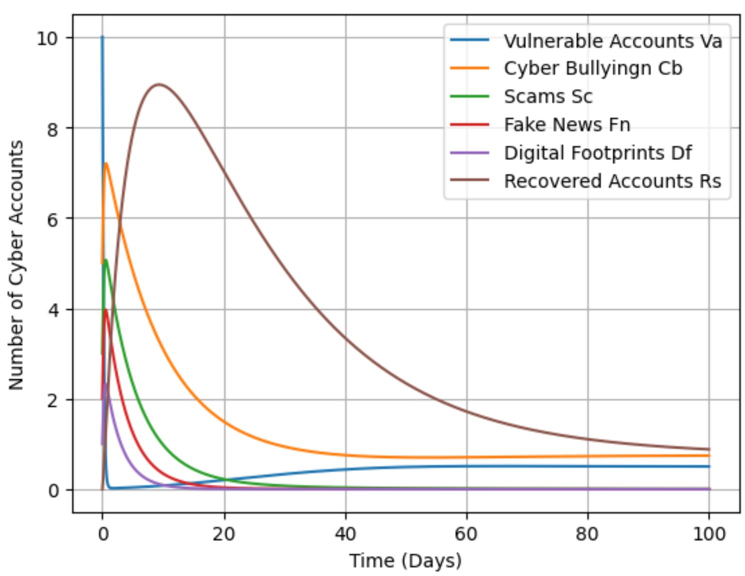

Figure 7. The behavior of the total cyber accounts model variables varied within 100 days at the cyber vices free equilibrium when $R_{cb} > 1$.

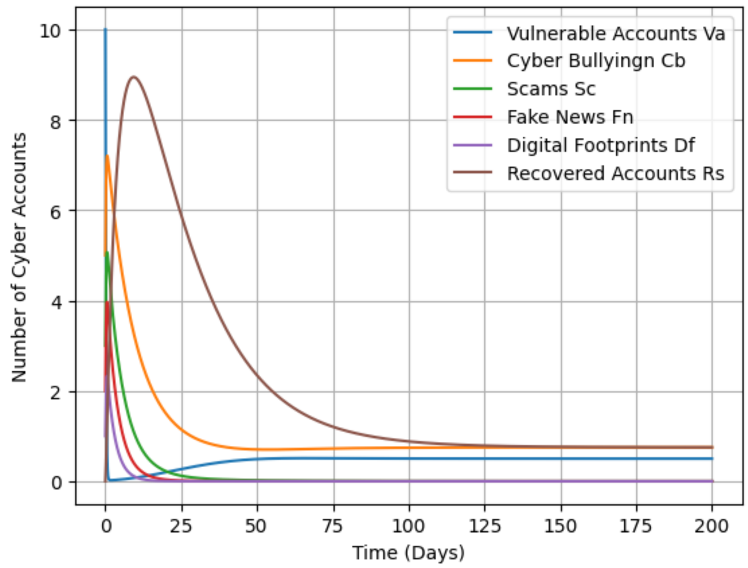

Figure 8. The behavior of the total cyber account model variables varied within 200 days at the cyber vices free equilibrium when $R_{cb} > 1$.

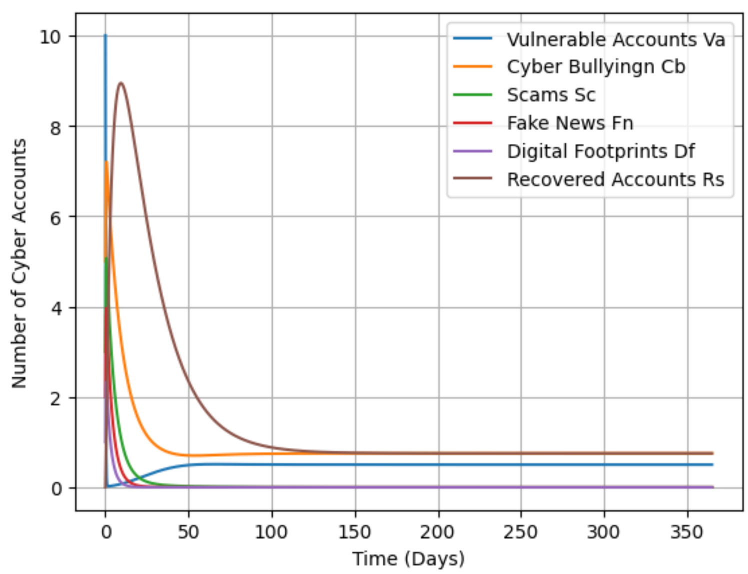

Figure 9. The behavior of the total cyber account model variables varied within 365 days at the cyber vices free equilibrium when $R_{cb} > 1$.

5. Conclusion

In this work, we have constructed a mathematical model aimed at understanding the behavior and spread of cyber vices within vulnerable online accounts and those engaged in activities such as bullying, scams, dissemination of fake news, and the creation of damaging digital footprints, along with examining the effects of relapse and recovery among these accounts. By applying appropriate mathematical theorems, we have verified the model's basic features, including its existence, uniqueness, positivity, and boundedness. Our investigations led us to identify equilibrium states and compute the basic reproduction number, R_{cb}, which is crucial for assessing the system's stability. Our findings suggest that when $R_{cb} < 1$, the cyber environment remains largely safe and stable, evidenced by the local and global asymptotic stability of the cyber vices-free equilibrium. On the other hand, if $R_{cb} > 1$, it indicates a tendency towards an endemic state of cyber vices within the cyberspace, characterized by local and global asymptotic stability, if left unchecked. Simulations conducted to corroborate the model's stability properties illustrate the potential for cyber vices to become endemic and intensify over time in the absence of effective mitigation strategies, thus posing a considerable threat to online safety. The implications of our work show the importance of implementing strategic interventions to counteract the spread of cyber vices and maintain a healthy and secure online environment. By devising and executing suitable control measures, it is possible to decrease the proliferation of malicious activities across cyber accounts, shield those at risk, and lessen the adverse effects of cyber threats. The equilibrium states, basic reproduction number R_{cb} and stability analysis provided by our model offer a crucial thought for the creation of focused and efficacious intervention strategies. It is imperative for policymakers, cyber security professionals, and other stakeholders to work together in formulating and applying proactive approaches to combat cyber vices, thereby preserving the integrity of the digital domain.

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