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A Comparative Analysis Between Adomian Decomposition Method and Differential Transformation Method for Solving Some Second-Order Ordinary Differential Equations

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Abstract The differential transformation method (DTM) and Adomian decomposition method (ADM) are two numerical methods that can be used to solve various differential equations. In this work, we compare the accuracy, convergence, and computational complexity of these two methods by using them to solve second-order nonlinear equations using a new differential operator for the second-order equation. We also used both methods to solve second-order nonlinear differential equations.

Keywords Differential transformation method, Adomian decomposition method, second-order ordinary differential equations

MSC(2010) 34Nxx, 34N05.

1. Introduction

Quantitative descriptions of many models in the physical, biological, and even social sciences are provided through the use of differential equations. These descriptions are usually made in terms of unknown functions of one, two, or more independent variables and relationships between the derivatives of these variables. If two or more independent variables are involved, the differential equation is called partial differential equation(PDE). Otherwise, it is called an ordinary ordinary differential equation(ODE) [28] . Modeling using differential equations is crucial as it provides relevant insights into the dynamics of many engineering and technical equipment and processes [20, 22]. However, many such models involve differential equations that are inherently nonlinear and difficult to solve. Many numerical methods have been developed to solve various differential equations that cannot be solved analytically [3]. But most numerical methods require discrimination, militarization. The rapid advancement of technology in today's ear has created an increasing need for scientific computing when processing and analyzing big date embodied in large amounts of real-life modeling phenomena. Numerical methods for solving nonlinear

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(ODEs) and (PDEs) are at the heart of many scientific calculations [21,30]. Differential equations have become a useful tool for describing these natural phenomena in scientific and engineering models. Therefore, it becomes important to be familiar with all traditional and recently developed methods for solving differential equations and their implementation. Although many standard methods exist for solving differential equations, more efficient methods still need to be developed or investigated [14]. In 2023 H. Chen and others studied a two-grid temporal second-order scheme for the two-dimensional nonlinear Voltmeter integro-differential equation with weakly singular kernel [13]. This year a study was conducted a computational technique for computing second-type mixed integral equations with singular kernels by A. M. S. Mahdy et al [18]. While AMR. Mahdy and others solved the fractional integro-differential equations using least squares and shifted Legender methods [19]. The main question of this research is which is better, the ADM or the DTM for solving second-order equations?

2. Adomian method

The Adomian decomposition method demonstrates rapid convergence of the solution and provides several significant advantages. This method was introduced and developed by George Adomian form the 1970s to the 1990s [2], as noted by Wen Jin and Yani [17].

Advantages of ADM: It is easy to understand and can be used ot solve many types of linear and nonlinear systems, such as algebraic equations, ordinary and partial differential equations, linear and nonlinear integral equations, differential equations, integral nonlinear stochastic operator equations, etc [10].

3. Analysis of Adomian decomposition method

From [23], we consider of second-order ordinary differential equations with constant coefficients of the form,

$$u'' + (m-2n)u' - n(m-n)u = q(x,u), \tag{3.1}$$
$$u(0) = A, u'(0) = B,$$

where $q(x,u)$ is a nonlinear function, A, B, n, m are constants. The differential operator, is as follows,

$$L(.) = e^{nx}\frac{d}{dx}e^{-mx}\frac{d}{dx}e^{(m-n)x}(.). \tag{3.2}$$

The inverse operator L^{-1} is therefore considered a quadratic integral operator as demonstrated below,

$$L^{-1}(.) = e^{-(m-n)x}\int_0^x e^{mx}\int_0^x e^{-nx}(.). \tag{3.3}$$

The Adomian decomposition method introduces the solution $u(x)$ and the nonlinear function $q(x,u)$ as infinite series,

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{3.4}$$

and

$$q(x,u)=\sum_{n=0}^{\infty}A_n, \tag{3.5}$$

where an are the formulate Adomian polynomials. The Adomian polynomials are formulated as follows:

$$A_0=F(u),$$
$$A_1=F'(u_0)u_1,$$
$$....$$

Now we get

$$\sum_{0}^{\infty}u_n(x)=\phi(x)+L^{-1}\sum_{0}^{\infty}A_n(x),$$

and the components u_n can be found as follows:

$$u_0=\phi(x)+L^{-1}q(x,u),$$
$$u_{n+1}=L^{-1}A_n, n\geq 0,$$

then

$$u_0=\phi(x)+L^{-1}q(x,u),$$
$$u_1=L^{-1}A_0,$$
$$u_2=L^{-1}A_1,$$
$$u_3=L^{-1}A_2.$$

4. Differential transformation method

Differential transformation method was first proposed by Zhou in 1986 [16]. So far, DTM has been developed in the literature to solve various differential and integral equations. For example Chen 1999 developed the DTM for solving partial differential equations [15], and Ayaz 2004 applied the method to differential algebraic equations [7]. In [4] Arikglu and Ozkol used DTM to solve integro-differential equations with boundary value conditions. Odibat used the DTM to solve volterra integral equations with separable kernels [5]. Cetinkaya and Kimaz used the generalized differential transform method for solving the time-fractional diffusion equations [11]. F. Ziyaee and A. Tari studied DTM for tow-dimensional Fredholm integral equations [29]. More recently, Farhana et al. Used DTM to solve third-order ordinary differential equations [8].

5. Analysis differential transformation method

The K-th derivative of a function with one variable is transformed into

$$U(k)=\frac{1}{k!}(\frac{d^k u(x)}{dx^k})atx=x_0, \tag{5.1}$$

where $u(x)$ is the original function, $U(K)$ is the transformation function , and the inverse differential transformation u(K) is defined as,

$$u(x) = \sum_{k=0}^{\infty} u(k)(x - x_0^k), \tag{5.2}$$

where $x_0 = 0$, and the function $u(x)$ defined in (5.2) is expressed as follows,

$$u(x) = \sum_{k=0}^{\infty} u(k)x^k. \tag{5.3}$$

Equation(5.3) shows the similarity between one-dimensional differential transformations and one-dimensional Taylor series expansions [1].

6. Fundamental operation performed by differential transformation method

The following table shows some of the basic performed using the DTM in [15].

Original functions	Transformed functions
$u(x) = g(x) \pm h(x)$	$U(k) = G(k) \pm H(k)$
$u(x) = cg(x)$	$U(k) = cG(k)$ where c is a constant
$u(x) = u'(x)$	$U(k) = (k+1)U(k)$
$u(x) = u''(x)$	$U(k) = (k+1)(k+2)U(k+2)$
$u(x) = x^m)$	$U(k) = \delta(k-m) = 1 if k = m, 0 if k \neq m$
$u(x) = u^m(x)$	$U(k) = (k+1)(k+2)..(k+m)U(k+m)$
$u(x) = e^{au}$	$U(k) = \frac{a^k}{k!}$

7. Comparison of two methods

In 2015, a comparison between the two methods was conducted to find the numerical solution of multi-pantograph delay differential equations [9], in 2019 R.B.Ogunrinde made a comparison between the ADM and DTM for solving first order ordinary differential equations [24]. N.H.Sekgothe made a comparison between two methods for solving ordinary and partial nonlinear differential equations [28]. In this paper we compare the two methods for solving second-order nonlinear differential equations, using a new differential operator. Through these studies we concluded that: the accuracy of DTM and ADM depends on the order of the differential equation and the number of terms used in the series. In general, DTM is more accurate than ADM for higher order differential equations. However, for low-order differential equations, ADM is more efficient than DTM [1]. The convergence of DTM and ADM depends on the properties of the differential equations. Often DTM converges more slowly than ADM [27].

8. Examples

In this section, we will give some examples to demonstrate the efficiency and accuracy of the proposed methods, and the convergence of them.

Example 1:

For $n = 3, m = 1$, the equation (3.1) is rewritten as ,

$$u'' - 5u' + 6u = 2e^x - e^{2x} + u^2, \tag{8.1}$$

$$u(0) = u'(0) = u''(0) = 1.$$

And give the exact solution,

$$u(x) = e^x.$$

Solving the Eq.(8.1) by ADM. The Eq.(8.1) is rewritten as,

$$L(u) = 2e^x - e^{2x} + u^2.$$

Writing the given differential equation in an operator form gives

$$L(.) = e^{3x}\frac{d}{dx}e^{-x}\frac{d}{dx}e^{-2x}(.).$$

With initial conditions,

$$u(0) = u'(0) = 1.$$

So L^{-1} is given by

$$L^{-1}(.) = e^{2x}\int_0^x e^x \int_0^x e^{-3x}(.).$$

We use L^{-1} for the Eq.(8.1) and get,

$$u(x) = \phi(x) + L^{-1}(2e^x - e^{2x}) + L^{-1}(u^2),$$

and

$$\phi(x) = 2e^{2x} - 2e^{3x}.$$

Then value number one for u is

$$u_0 = \phi(x) + L^{-1}(2e^x - e^{2x}),$$

$$u_0 = 1 + x - x^3 - \frac{4x^4}{3} - \frac{13x^5}{12} - \frac{59x^6}{90} - \frac{23x^7}{72}. \tag{8.2}$$

And the nonlinear part is

$$u_{n+1} = L^{-1}(A_n), n \geq 0,$$

$$u_1 = \frac{x^2}{2} + \frac{7x^3}{6} + \frac{31x^4}{24} + \frac{101x^5}{120} + \frac{23x^6}{80} - \frac{151x^7}{5040}, \tag{8.3}$$

$$u_2 = \frac{x^4}{12} + \frac{x^5}{4} + \frac{16x^6}{45} + \frac{373x^7}{1260}. \tag{8.4}$$

Then,

$$u(x) = u_0 + u_1 + u_2,$$

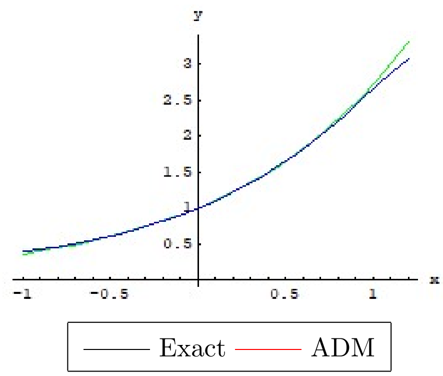

Figure 1. The exact solution $u = e^x$ and the ADM solution $u = \sum_{n=0}^{2} u_n(x)$.

$$u(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{24} + \frac{x^5}{120} - \frac{x^6}{80} + \frac{269x^7}{5054}. \quad (8.5)$$

From the following figure, we notice the solution obtained using the Adomian Decomposition Method ADM closely approximates the exact solution.

Solving the equation(8.1) by DTM. From the initial condition and theorem, we obtain,

$$U(0) = 1, U(1) = 1, \quad (8.6)$$

$$U(k+2) = \frac{k!}{(k+2)!}(5(k+1)U(k+1) - 6U(k) + 2\frac{1}{k!} - \frac{2^{k!}}{k!} + \sum_{r=0}^{k} U(r)U(k-r)). \quad (8.7)$$

Substituting Eq.(8.6) into Eq.(8.7) at $k = 0$, we have:

$$U(2) = \frac{1}{2}. \quad (8.8)$$

Using the recurrence relation Eq.(8.1) at k=1,2...., we obtain:

$$U(3) = \frac{1}{6}.$$

We can write the solution as,

$$u(x) = \sum_{k=0}^{\infty} U(k)x^k. \quad (8.9)$$

Hence,

$$u(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + ... \quad (8.10)$$

Here we notice a remarkable convergence between the DTM solution and the complete solution.

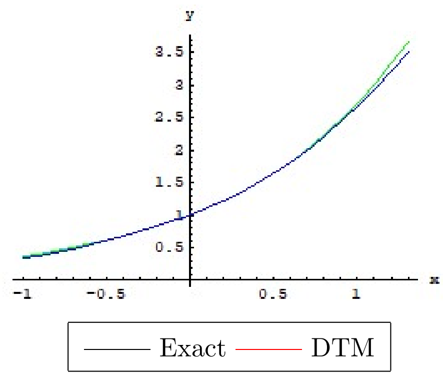

Figure 2. The exact solution $u = e^x$ and the DTM solution $u = \sum_{k=0}^{\infty} U(k)x^k$.

x	DTM	ADM	Absolute error
0.0	1	1	0
0.1	1.10517	1.10517	0.00000
0.2	1022133	1.2214	0.00007
0.3	1.3495	1.34984	0.00034
0.4	1.49067	1.491613	0.00096
0.5	1.64583	1.64809	0.00226
0.6	1.8816	1.81997	0.00397
0.7	2.00217	2.00771	0.00554
0.8	2.20533	2.21066	0.00533
0.9	2.4265	2.42659	0.00009

From the table we notice the convergence of solutions between ADM and DTM. From the figure, we notice that the solutions of the two methods are close to the complete solution.

Example 2:
By comparing Eq(3.1). We get $n = 0, m = 1$ and rewritten as,

$$u'' + u' = 1 + 2e^x - (x + e^x)^2 + u^2, \tag{8.11}$$

$$u(0) = 1, u'(0) = 2.$$

The exact solution is,

$$u(x) = x + e^x.$$

And solving it by ADM. The Eq.(8.11) is rewritten as,

$$L(u) = 1 + 2e^x - (x + e^x)^2 + u^2.$$

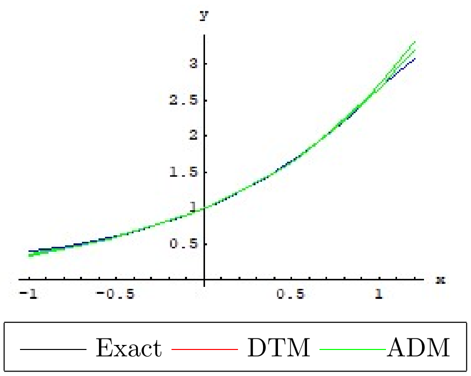

Figure 3. The exact solution $u = e^x$ and the DTM solution $u = \sum_{k=0}^{\infty} U(k)x^k$and the ADM solution $u = \sum_{n=0}^{2} u_n(x)$.

Writing the given differential equation in an operator form gives,

$$L(.) = \frac{d}{dx}e^{-x}\frac{d}{dx}e^{x}(.).$$

So L^{-1} is given by,

$$L^{-1}(.) = e^{-x}\int_0^x e^x \int_0^x (.).$$

We use L^{-1} for the Eq.(8.11) and get,

$$u(x) = \phi(x) + L^{-1}(1 + 2e^x - (x + e^x)^2) + L^{-1}(u^2),$$

and,

$$\phi(x) = 3 - 2e^{-x}.$$

Then value number one for u is

$$u_0 = \phi(x) + L^{-1}(1 + 2e^x - (x + e^x)^2),$$

$$u_0 = 1 + 2x + \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{20}. \tag{8.12}$$

And the nonlinear part is

$$u_{n+1} = L^{-1}(A_n), n \geq 0,$$

$$u_1 = \frac{x^2}{2} + \frac{x^3}{2} + \frac{5x^4}{24} - \frac{3x^5}{40}, \tag{8.13}$$

$$u_2 = \frac{x^4}{12} + \frac{2x^5}{15} + \frac{7x^6}{120} - \frac{53x^8}{3360}. \tag{8.14}$$

Then,

$$u(x) = u_0 + u_1 + u_2,$$

$$u(x) = 1 + 2x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120} + \frac{7x^6}{120} - \frac{53x^8}{3360}. \tag{8.15}$$

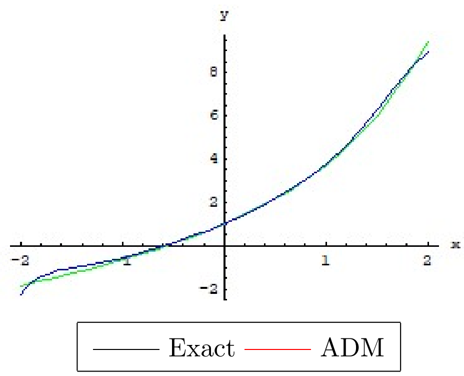

Figure 4. The exact solution $u = x + e^x$ and the ADM solution $u = \sum_{n=0}^{\infty} u_n(x)$.

Here we notice that the solution by ADM is close to the complete solution.

Solving Eq.(8.11) by DTM. Taking the DTM of Eq.(8.11) and the initial condition respectively, we obtain

$$U(k+2) = \frac{k!}{(k+2)!}(-(k+1)U(k+1) + \delta(k) + 2\frac{1^k}{k!} - \delta(k-2) - 2\sum_{r=0}^{k}\frac{\delta(k-1)}{k!}$$

$$-\frac{2^k}{k!} + \sum_{r=0}^{k} U(r)U(k-r)). \tag{8.16}$$

Using the initial condition, we have

$$U(0) = 1, U(1) = 2.$$

If $k = 0$ we have,

$$U(2) = \frac{1}{2}.$$

Using the recurrence relation Eq.(8.11) at k=1,2...., we obtain:

$$U(3) = \frac{1}{3}.$$

We can write the solution as,

$$u(x) = \sum_{k=0}^{\infty} U(k)x^k. \tag{8.17}$$

Hence,

$$u(x) = 1 + 2x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + ... \tag{8.18}$$

This figure shows convergence of the solution by DTM and the complete solution.

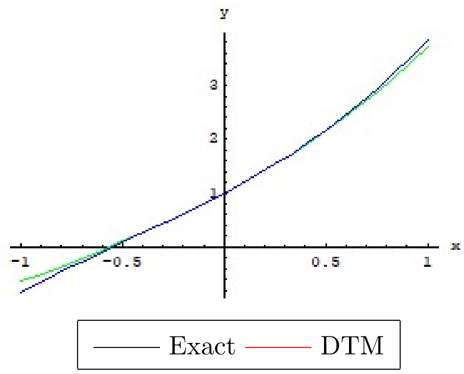

Figure 5. The exact solution $u = x + e^x$ and the DTM solution $u = \sum_{k=0}^{\infty} U(k)x^k$.

x	DTM	ADM	Absolute error
0	1	1	0
0.1	1.20533	1.020517	0.00016
0.2	1.42267	1.42141	0.00126
0.3	1.654	1.6499	0.0041
0.4	1.90133	1.89205	0.00928
0.5	2.1667	2.14955	0.01715
0.6	2.452	2.4245	0.0275
0.7	2.75933	2.71952	0.03981
0.8	3.09067	3.03778	0.05289
0.9	3.448	3.38297	0.06503

This table shows the convergence between the solutions of the ADM and DTM.

This figure shows the convergence between the solutions of ADM and DTM.

Example 3:

By comparing Eq.(3.1), when $n = 1, m = -1$, it is rewritten as,

$$u'' - 3u' + 2u = 2x - 3 - x^2 + u^2, \tag{8.19}$$

$$u(0) = 0, u'(0) = 1.$$

And the exact solution is,

$$u(x) = x.$$

And solve it by AMA. The Eq.(8.19) is rewritten as,

$$L(u) = 2x - 3 - x^2 + u^2.$$

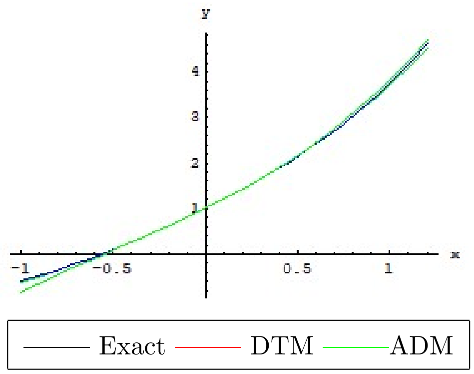

Figure 6. The exact solution $u = e^x$ and the DTM solution $u = \sum_{k=0}^{\infty} U(k)x^k$ and the ADM solution $u = \sum_{n=0}^{2} u_n(x)$.

Writing the given differential equation in an operator form gives,

$$L(.) = e^x \frac{d}{dx} e^x \frac{d}{dx} e^{-2x}(.).$$

So L^{-1} is given by,

$$L^{-1}(.) = e^{2x} \int_0^x e^{-x} \int_0^x e^{-x}(.).$$

We use L^{-1} for the Eq.(8.19) and get,

$$u(x) = \phi(x) + L^{-1}(2x - 3 - x^2 + u^2),$$

and,

$$\phi(x) = e^{2x} - e^x.$$

Then value number one for u is

$$u_0 = \phi(x) + L^{-1}(2x - 3 - x^2),$$

$$u_0 = x - \frac{x^4}{12} - \frac{x^5}{20} - \frac{7x^{360}}{16} - \frac{x^7}{168}. \tag{8.20}$$

And the nonlinear part is

$$u_{n+1} = L^{-1}(A_n), n \geq 0,$$

$$u_1 = \frac{x^4}{1}2 + \frac{x^5}{20} + \frac{7x^6}{360} + \frac{x^7}{504}, \tag{8.21}$$

$$u_2 = \frac{x^7}{252} + \frac{11x^8}{3360} + \frac{23x^9}{15120}. \tag{8.22}$$

Then,

$$u(x) = u_0 + u_1 + u_2,$$

$$u(x) = x + \frac{11x^8}{3360} + \frac{23x^9}{15120}. \tag{8.23}$$

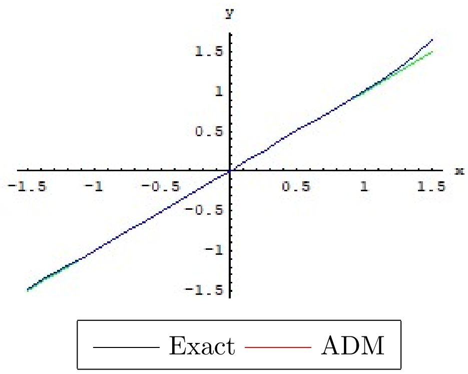

Figure 7. The exact solution $u = x$ and the ADM solution $u = \sum_{n=0}^{2} u_n(x)$.

The figure shows the convergence of the solution by ADM and the complete solution.

Solving Eq.(8.19) by DTM. Taking DTM of Eq.(8.19) and the initial condition respectively, we obtain

$$U(k+2) = \frac{k!}{(k+2)!}\,(3(k+1)U(k+1) - 2U(k) + 2\delta(k-1) - 3\delta(k) \tag{8.24}$$
$$-\delta(k-2) + \sum_{r=0}^{k} U(r)U(k-r)\,).$$

Using the initial condition, we have

$$U(0) = 0, U(1) = 1.$$

If $k = 0$ we have,

$$U(2) = \frac{1}{2}.$$

Using recurrence relation Eq.(8.19) at k=1,2...., we obtain:

$$U(3) = \frac{1}{2}.$$

We can write the solution as,

$$u(x) = \sum_{k=0}^{\infty} U(k)x^k. \tag{8.25}$$

Hence,

$$u(x) = x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + ... \tag{8.26}$$

The solution graph show the convergence of the (DTM) and the complete solution.

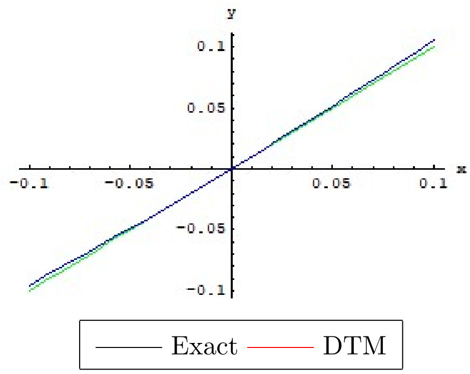

Figure 8. The exact solution $u = x$ and the DTM solution $u = \sum_{k=0}^{\infty} U(k)x^k$.

x	DTM	ADM	Absolute error
0	0	0	0
0.1	0.1055	0.1	0.0055
0.2	0.224	0.2	0.024
0.3	0.35850	0.30585	0.0526
0.4	0.512	0.00003	0.111997
0.5	0.6875	0.500016	0.187484
0.6	0.888	0.60007	0.28793
0.7	1.1165	0.70025	0.41625
0.8	1.376	0.800753	0.424753
0.9	1.6695	0.901999	0.767501

The previous table shows the convergence of the solutions of the two methods.

The previous figure shows the convergence of the solutions of the two methods with the complete solution.

9. Conclusion

In this work, we compare the accuracy and convergence of ADM and DTM for solving second-order nonlinear differential equations to enhance the solution process, we introduce a new differential operator and apply it within both methods. We also present examples of second-order equations to demonstrate how both methods can be used. ADM and DTM are powerful numerical methods that can be used to solve a variety of differential equations. Both methods have proven their accuracy in solving such equations.

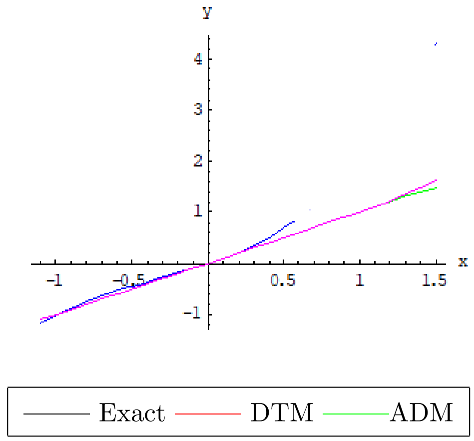

Figure 9. The exact solution $u = x$ and the DTM solution $u = \sum_{k=0}^{\infty} U(k)x^k$ and the ADM solution $u = \sum_{n=0}^{2} u_n(x)$.

10. Results and recommendations

1- Both ADM and DTM are considered accurate for solving differential equations of all types.
2- The ADM is more accurate in solving, while the DTM easier in finding the solution.
3- The differential operator in this research can be used in many similar equations with different values of constants.
4- Both methods have satisfactory results that closely approximate the complete solution.

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