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Parabolic System Related to the P-Laplician with Degeneracy on the Boundary

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Abstract. In this article, we study the system with boundary degeneracy

$$u_{it}-\mathrm{div}(a(x)|\nabla u_i|^{p_i-2}\nabla u_i)=f_i(x,t,u_1,u_2), \qquad (x,t)\in\Omega_T.$$

Applying the monotone iterattion technique and the regularization method, we get the existence of solution for a regularized system. Moreover, under an integral condition on the coefficient function $a(x)$, the existence and the uniqueness of the local solutions of the system is obtained by using a standard limiting process. Finally, the stability of the solutions is proved without any boundary value condition, provided $a(x)$ satisfies another restriction.

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1 Introduction

Let $\Omega\subset R^n$ be a bounded domain with suitable smooth boundary $\partial\Omega$. We study the parabolic system

$$u_{it}-\mathrm{div}(a(x)|\nabla u_i|^{p_i-2}\nabla u_i)=f_i(x,t,u_1,u_2), \qquad (x,t)\in\Omega_T \tag{1.1}$$

$$u_i(x,0)=u_{i0}(x), \qquad x\in\Omega, \tag{1.2}$$

$$u_i(x,t)=0, \qquad (x,t)\in\Gamma_T, \tag{1.3}$$

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where $p_i>2$, $i=1,2$. $\Omega_T=\Omega\times(0,T),\Gamma_T=\partial\Omega\times(0,T)$.

When $a(x)$=1, system (1.1) is the usual parabolic system

$$u_{it}-\mathrm{div}(|\nabla u_i|^{p_i-2}\nabla u_i)=f_i(x,t,u_1,u_2). \tag{1.4}$$

If $f_i=0$, one equation of the system (1.1) is the usual evolutionary p-Laplacian eqution

$$u_t-\mathrm{div}(|\nabla u|^{p-2}\nabla u)=0. \tag{1.5}$$

The equation (1.5) was arised from many applications in the fields of mechanics, physics and biology [1-9]. For example, in the non-Newtonian fluids theory, p is a characteristic quantity of the medium. We call them dilatant fluids when $p>2$ and those pseudoplastics when $p<2$. Certainly, if $p=2,u_t=\Delta u$, it is the Newtonian fluid equation. More generally, it is well-known as the classical heat conduction equation. The solution of equation has infinite propagation speeds of disturbance, which seems unreasonable. Therefore, it's better to reflect the actual physical situation for equation (1.5) when $p\neq 2$. Particularly, the solution of equation has finite propagation speeds of disturbance when $p>2$ [5].

There are abundant articles on the global finiteness and blow-up properties of the solutions. With various boundary conditions to the equations and systems of evolutionary p-Laplacian equations, we can refer to [10-18]. In [19], Wei and Gao researched the existence and uniqueness of solutions to (1.4) with the condition (1.2-1.3). In [20], Zhao investigated the existence and blowing-up properties of the solutions for some single equation.

It is in general difficult to research the system since the system is coupled with nonlinear terms. In this paper, we study the influence on the solutions of the system (1.1) due to $a(x)|_{x\in\partial\Omega}=0$. Firstly, with the help of regularization method and monotone iteration technique, we obtain the existence of solutions to a regularized system of equations. And then using a standard limiting process, we obtain the existence and unique of the local solutions of the problem (1.1) provided $a(x)$ satisfies some restriction.

The system (1.1) degenerates when $u_i=0$, $\nabla u_i=0$ or $a(x)=0$. In general, there are no classical solutions and hence we study the generalized solutions to the problem (1.1)-(1.3). As preliminaries, we first introduce some definitions as follows.

Definition 1.1. *A function $u=(u_1,u_2)$ is called a generalized solution of the system* (1.1)-(1.2) *if $u_i\in L^\infty(\Omega_T)\cap L^{p_i}(0,T;W_0^{1,p_i}(\Omega))$, $u_{it}\in L^2(\Omega_T)$, and satisfies*

$$\int\int_{\Omega_T}(u_{it}\varphi_i+a(x)|\nabla u_i|^{p_i-2}\nabla u_i\nabla\varphi_i)\mathrm{d}x\mathrm{d}t=\int\int_{\Omega_T}f_i(x,t,u_1,u_2)\varphi_i\mathrm{d}x\mathrm{d}t, \tag{1.6}$$

for any $\varphi_i\in C^1(\overline{\Omega_T}),\varphi_i(x,T)=0,\varphi_i(x,t)=0,$ for $(x,t)\in\Gamma_T,i=1,2$.