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Stochastic Differential Equations Driven by Multi-fractional Brownian Motion and Poisson Point Process

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Abstract. In this paper, we study a class of stochastic differential equations with additive noise that contains a non-stationary multifractional Brownian motion (mBm) with a Hurst parameter as a function of time and a Poisson point process of class (QL). The differential equation of this kind is motivated by the reserve processes in a general insurance model, in which between the claim payment and the past history of liability present the long term dependence. By using the variable order fractional calculus on the fractional Wiener-Poisson space and a multifractional derivative operator, and employing Girsanov theorem for multifractional Brownian motion, we prove the existence of weak solutions to the SDEs under consideration, As a consequence, we deduce the uniqueness in law and the pathwise uniqueness.

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1 Introduction

Recently, Bai and Ma [2] studied the following stochastic differential equation:

$$X_t = x + \int_0^t b(s,X_s)\mathrm{d}s + B_t^H - L_t, \quad t\in[0,T], \tag{1.1}$$

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where $B^H = \{B_t^H : t \geq 0\}$ is a fractional Brownian motion (fBm) with Hurst parameter $H \in (0,1)$, defined on a given filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{F})$, with $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$ being a filtration that satisfies the usual hypotheses (cf., e.g., [18]); and $L = \{L_t : t \geq 0\}$ is a Poisson point process of class (QL), independent of B^H. More precisely, we assume that L takes the form

$$L_t = \int_0^t \int_{\mathbb{R}} f(s,x) N_p(\mathrm{d}s, \mathrm{d}x), \quad t \geq 0,$$

where f is a deterministic function, and p is a stationary Poisson point process whose counting measure N_p is a Poisson random measure with Lévy measure ν.

The fBm B^H possesses long-range dependence, which has been noted in insurance models based on the observations that the claims often display long memories due to extreme weather, natural disasters, and also noted in casualty insurance such as automobile third-party liability (cf. e.g., [5–7, 9, 14] and references therein). A perturbed by fBm reserve (or surplus) model is of the following form:

$$X_t = x + c(1+\rho)t + \varepsilon B_t^H - L_t, \quad t \in [0,T].$$

Here $x \geq 0$ denotes the initial surplus, $c > 0$ is the premium rate, $\rho > 0$ is the "safety" (or expense) loading, $\varepsilon > 0$ is the perturbation parameter, L_t is a Poisson point process of class and denotes cumulated claims up to time t, and $T > 0$ is a fixed time horizon. In fact, if we assume further that in addition to the premium income, the company also receives interest of its reserves at time with interest rate $r > 0$, and that the safety loading ρ also depends on the current reserve value, one can argue that the reserve process X should satisfy an SDE of the form of (1.1) with

$$b(t,x) = rx + c(1 + \rho(t,x)), \quad (t,x) \in [0,T] \times \mathbb{R}.$$

However, the Hurst parameter H of the fractional Brownian motion can be dependent on time (e.g. [17]). The process was named multifractional Brownian motion, referring to the fact that the fractional parameter H was a function depending on time taking values between 0 and 1. As showed in the literature (see [13, 17]), multifractional Brownian motion seems to be a more flexible model than fractional Brownian motion. The multifractional Brownian motion possesses the good feature of fractional Brownian motion, such as Hölder continuity (see [4,19]); self-similarity (see [15]) and long rang dependence (see [1]) etc. But, the multifractional Brownian motion is a non-stationary stochastic process which makes it more intricate to deal with stochastic differential equations driven by multifractional Brownian motion.

In connection with the above discussions, in this paper, we are interested in the following stochastic differential equation (SDE):

$$X_t = x + \int_0^t b(s, X_s) \mathrm{d}s + B_t^h - L_t, \quad t \in [0,T], \tag{1.2}$$