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Eigenvalues of Elliptic Systems for the Mixed Problem in Perturbations of Lipschitz Domains with Nonhomogeneous Neumann Boundary Conditions

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Abstract. We study eigenvalues of an elliptic operator with mixed boundary conditions on very general decompositions of the boundary. We impose nonhomogeneous conditions on the part of the boundary where the Neumann term lies in a certain Sobolev or L^p space. Our work compares the behavior of and gives a relationship between the eigenvalues and eigenfunctions on the unperturbed and perturbed domains, respectively.

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1 Introduction

The literature contains much analysis on the study of eigenvalues for elliptic equations, but work on systems of equations is much more scarce. Many of the estimates used for equations do not hold for systems and thus analysis for systems requires something different. Moreover, we use the reverse Hölder technique frequently to achieve our estimates. In this paper, we look at the behavior of eigenvalues and eigenfunctions on perturbed domains and compare them to ones on the unperturbed domain. We give a simple characterization of families of perturbed domains, which include dumbbell shaped domains, but these families may be quite general. We work in Lipschitz domains and assume that

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the coefficients of the operator are bounded and symmetric. Furthermore, we assume that the Dirichlet set D satisfies a corkscrew condition, which allows for a rather general decomposition of the boundary. A term which lies in a Sobolev space or L^p space is imposed on the Neumann set N.

There are many results on the study of eigenvalues for equations on perturbed domains when we have Dirichlet boundary conditions. A classic paper by Babuska and Výborný [1] shows continuity of Dirichlet eigenvalues for elliptic equations under a regular variation of the domain. Work by Davies [2] and Pang [3] studies the relationship between Dirichlet eigenvalues and corresponding eigenfunctions in a domain Ω and eigenvalues and eigenfunctions in sets of the form $R(\varepsilon) = \{x \in \Omega : \mathrm{dist}(x,\partial\Omega) \geq \varepsilon\}$. Two papers by Chavel and Feldman [4] and Anné and Colbois [5] examine eigenvalues on compact manifolds with a small handle and Dirichlet conditions on the ends of the handle. More recent work for Dirichlet conditions includes work by Daners [6], which shows convergence of solutions to elliptic equations on sequences of domains. Burenkov and Lamberti [7] prove spectral estimates for higher-order elliptic operators on domains in certain Hölder classes. Kozlov [8] gives asymptotics of Dirichlet eigenvalues for domains in $\mathbb{R}^n$ and Grieser and Jerison [9] also give asymptotics for Dirichlet eigenvalues and eigenfunctions on plane domains.

When a Neumann condition is placed on part of the boundary, the eigenvalue problem is much more difficult to analyze. A classic example by Courant and Hilbert [10] shows that continuity of eigenvalues is not generally obtained for Neumann eigenvalues if the domain is only C^0. In fact, Arrieta, Hale, and Han [11] show that if the domain is not sufficiently smooth, none of the Neumann eigenvalues $\{\lambda_m^\varepsilon\}$ converge for $m \geq 3$. However, if one places more regularity on the domains, rates of convergence are achievable. This is illustrated in work by Jimbo [12], Jimbo and Kosugi [13], and Brown, Hislop, and Martinez [14].

As mentioned earlier, there seems to be a lot less work on the study of perturbed domains with systems of equations. Fang [15] studied the behavior of the second eigenvalue in a perturbed domain for a system of equations in $\mathbb{R}^2$. Taylor [16] provided rates of convergence for Dirichlet eigenvalues involving elliptic systems on domains with low regularity. More recent work by Collins and Taylor [17] showed convergence of eigenvalues for the mixed problem with homogeneous Dirichlet and Neumann boundary conditions. The contribution in this paper continues the study of eigenvalues for these types of operators for the mixed problem when the Neumann term is nontrivial.

2 Preliminaries and main results

We will be working on a bounded Lipschitz domain Ω. This means that locally on the boundary, Ω is a domain which lies above the graph of a Lipschitz function. In order to give a formal definition, we introduce coordinate cylinders. Given a constant $M > 0$, $x \in \partial\Omega$, and $r > 0$, we define a coordinate cylinder $Z_r(x) = \{y : |y' - x'| < r, |y_n - x_n| <$