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Global Regularity of 2D Leray-Alpha Regularized Tropical Climate Models

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Abstract. In this paper, we establish the global regularity of 2D leray-alpha regularized tropical climate models. The global strong solution to the system with a half Laplacian of the first baroclinic model of velocity (Λv) and thermal diffusion $(-\Delta\theta)$ or with only the dissipation of the barotropic mode $(-\Delta u)$ are obtained.

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Key Words: Tropical climate model; Leray-alpha model; global regularity.

1 Introduction

The 2D tropical climate model with fractional dissipation can be written as:

$$\begin{cases} \partial_t u+u\cdot\nabla u+\mu\Lambda^{2\alpha}u+\nabla\cdot(v\otimes v)=0, \\ \partial_t v+u\cdot\nabla v+\nu\Lambda^{2\beta}v+v\cdot\nabla u+\nabla\theta=0, \\ \partial_t\theta+u\cdot\nabla\theta+\eta\Lambda^{2\gamma}\theta+\nabla\cdot v=0, \\ \nabla\cdot u=0, \end{cases} \tag{1.1}$$

for $t\geq 0$, $x\in\mathbb{R}^2$. We denote $u=(u_1(x,t),u_2(x,t))$, $v=(v_1(x,t),v_2(x,t))$ the barotropic mode and the first baroclinic mode of the velocity, respectively, and the scalars θ and p represent the temperature and the pressure. μ, ν, η, α, γ, $\beta\geq 0$ are real parameters. Here $v\otimes v$ is the standard tensor notation and the fractional Laplacian operator $\Lambda^{2\alpha}$ is defined via the Fourier transform

$$\widehat{\Lambda^{2\alpha}f}(\xi)=|\xi|^{2\alpha}\widehat{f}(\xi).$$

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Let us briefly recall some works on the tropical climate model (1.1) firstly. When $\alpha=\beta=1$, $\gamma=0$, Li and Titi [1] established the global well-posedness of strong solutions. For the case with the very weak barotropic dissipation $(\alpha>0)$, Ye obtained the global smooth solution when $\beta=\gamma=1$ in [2], which is motivated by the generalized MHD system [3]. Later, Dong et al. [4] considered the global regularity for the 2D tropical climate model without thermal diffusion when $\alpha+\beta=2$, $1<\beta\leq\frac{3}{2}$. And in [5], Dong et al. focused on the case of $\mu=0$, the system (1.1) has a unique global classical solution when $\beta>1$, $\beta+\gamma>\frac{3}{2}$ or $\eta=0$, $\frac{3}{2}<\beta\leq 2$. In [6], Dong et al. establish the global existence and regularity of smooth solution for two ranges of the parameters α and γ when $\beta=1$. Recently, When $\alpha+\beta\geq 2$, $1<\alpha<2$, Ye [7] studied the global regularity problem of 2D tropical climate model with zero thermal diffusion, also established the global regularity results with logarithmically supercritical dissipation.

It should be pointed out that the system (1.1) reduces to the 2D MHD-type equations when $\theta=$ constant. The global regularity problem for the 2D MHD equation has attracted considerable attention (see, e.g, [3,8–15]). However, this is still an open problem whether or not there exists a global solution to the system (1.1) with $\mu=0$, $\nu>0$, $\beta=1$ or $\nu=0$, $\mu>0$, $\alpha=1$ when $\theta=$ constant. To bypass this difficulty, authors considered lots of models, like the so-called Leray-alpha models (see, e.g, [16–19]). In this direction, the purpose of this paper is to obtain further understanding of the global regularity problem of 2D tropical climate model with $\beta=\frac{1}{2},\gamma=1,\mu=0,\nu>0,\eta>0$ or $\alpha=1,\mu>0,\nu=\eta=0$, which can be expressed as

$$\begin{cases}\partial_t u+w\cdot\nabla u+\nabla p+\nabla\cdot(v\otimes v)=0, \;\; x\in\mathbb{R}^2,\ t>0,\\ \partial_t v+w\cdot\nabla v+\nu\Lambda v+v\cdot\nabla w+\nabla\theta=0,\\ \partial_t\theta+w\cdot\nabla\theta-\eta\Delta\theta+\nabla\cdot v=0,\\ u=w-a^2\triangle w,\\ \nabla\cdot w=\nabla\cdot u=0,\\ u(x,0)=u_0(x),v(x,0)=v_0(x),\theta(x,0)=\theta_0(x),\end{cases}\tag{1.2}$$

$$\begin{cases}\partial_t u+w\cdot\nabla u-\mu\Delta u+\nabla p+\nabla\cdot(v\otimes v)=0, \;\; x\in\mathbb{R}^2,\ t>0,\\ \partial_t v+w\cdot\nabla v+v\cdot\nabla w+\nabla\theta=0,\\ \partial_t\theta+w\cdot\nabla\theta+\nabla\cdot v=0,\\ u=w-a^2\triangle w,\\ \nabla\cdot w=\nabla\cdot u=0,\\ u(x,0)=u_0(x),v(x,0)=v_0(x),\theta(x,0)=\theta_0(x),\end{cases}\tag{1.3}$$

where $w=(w_1(x),w_2(x))$ denotes the "filtered" velocity, $u_0(x),v_0(x)$ and $\theta_0(x)$ are the given initial data satisfying $\nabla\cdot u_0=0$, $a>0$ is the length scale parameter that stands for the width of the filter. The system (1.2) and (1.3) are both the Leray-α model, which