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Solving System of Conformable Fractional Differential Equations by Conformable Double Laplace Decomposition Method

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Abstract. Herein, an approach known as conformable double Laplace decomposition method (CDLDM) is suggested for solving system of non-linear conformable fractional differential equations. The devised scheme is the combination of the conformable double Laplace transform method (CDLTM) and, the Adomian decomposition method (ADM). Obtained results from mathematical experiments are in full agreement with the results obtained by other methods. Furthermore, according to the results obtained we can conclude that the proposed method is efficient, reliable and easy to be implemented on related many problems in real-life science and engineering.

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1 Introduction

Recently, many researchers have been attracted by the fractional partial differential equations, and the topic has been received more attention during the last decades due to their significant role in engineering and real-life science [1–3]. Consequently, many authors introduced several classes of fractional derivatives such as, Caputo, Riemann-Liouville, Hadamard, Caputo-Hadamard and so on [4, 5]. Unfortunately, these fractional derivatives do not obey a lot of the usual properties such as the product rule, the chain rule,

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Quotient Rule for two functions, due to that, these fractional derivatives have a lot of difficulties in applications, to overcome these difficulties, Khalil et.al. [6] introduced the conformable fractional derivative that satisfying all the classical properties of the normal derivatives. Consequently, some techniques for solving ordinary differential equations are used to solve conformable fractional differential equations. Recently, many authors have developed various analytical and approximate methods to obtain the solution of partial differential equations, such as Homotopy perturbation method (HPM) [7,8], Adomian decomposition method (ADM) [9, 10], variational iteration method (VIM) [11], differential transform method (DTM) [12], transform methods [13–19], and many others. Among all the previous methods, the double Laplace method (DLM). In the last few years there was no work or very little work available on (DLT), therefore, recently many authors have been paying a significant attention towards the applying of double Laplace transform to solve integral, partial differential equations including ordinary and fractional [20–22]. Recently, Ozan Ozkan and Ali kurt in (2018) [23] introduced the conformable double Laplace transform (CDLT) and they implemented it to solve conformable fractional heat equation and Conformable fractional Telegraph equation. For more about (CDLTM) see [24, 25].

The main target of this article is to solve systems of nonlinear fractional differential equations involving conformable fractional derivatives by a new method called conformable double Laplace decomposition method, this method is a combination between two methods, which are the (CDLTM) and (ADM).

2 Preliminaries

In this section, properties and some basic definitions of the conformable fractional derivative (CFD) are presented.

Definition 2.1. ([26]) *The (CFD) of order ρ of a function $\varphi:(0,\infty)\to\mathbb{R}$ is given by:*

$$D_x^{\rho}\varphi\left(\frac{x^{\rho}}{\rho}\right)=\lim_{\varepsilon\to 0}\frac{\varphi\left(\frac{x^{\rho}}{\rho}+\varepsilon x^{1-\rho}\right)-\varphi\left(\frac{x^{\rho}}{\rho}\right)}{\varepsilon},\qquad \frac{x^{\rho}}{\rho}>0,\ 0<\rho\leqslant 1. \tag{2.1}$$

Definition 2.2. ([27]) *The (CFD) of order ρ of a function $\varphi\left(\frac{x^{\rho}}{\rho},\frac{\tau^{\beta}}{\beta}\right):\mathbb{R}\times(0,\infty)\to\mathbb{R}$ is defined by:*

$$D_x^{\rho}\varphi\left(\frac{x^{\rho}}{\rho},\frac{\tau^{\beta}}{\beta}\right)=\lim_{\varepsilon\to 0}\frac{\varphi\left(\frac{x^{\rho}}{\rho}+\varepsilon x^{1-\rho},\frac{\tau^{\beta}}{\beta}\right)-\varphi\left(\frac{x^{\rho}}{\rho},\frac{\tau^{\beta}}{\beta}\right)}{\varepsilon},\qquad 0<\rho,\ \beta\leqslant 1,\ \frac{x^{\rho}}{\rho},\frac{\tau^{\beta}}{\beta}>0. \tag{2.2}$$