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Fractal Dimension of Random Attractors for Non-autonomous Fractional Stochastic Reaction-diffusion Equations

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Abstract. This paper deals with non-autonomous fractional stochastic reaction-diffusion equations driven by multiplicative noise with $s\in(0,1)$. We first present some conditions for estimating the boundedness of fractal dimension of a random invariant set. Then we establish the existence and uniqueness of tempered pullback random attractors. Finally, the finiteness of fractal dimension of the random attractors is proved.

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1 Introduction

In this paper, we will discuss the following non-autonomous fractional stochastic reaction-diffusion equation with multiplicative noise on a bounded domain $U\subset\mathbf{R}^{\mathbf{n}}$:

$$\frac{\partial u}{\partial t}+(-\Delta)^s u+f(u)=g(t,x)+\alpha u\circ\frac{\mathrm{d}W}{\mathrm{d}t},\qquad x\in U,\, t>\tau, \tag{1.1}$$

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with boundary condition

$$u(t,x)=0, \qquad x\in\mathbf{R^n}\setminus U,\ t>\tau, \tag{1.2}$$

and initial condition

$$u(\tau,x)=u_\tau(x), \qquad x\in U, \tag{1.3}$$

where $s\in(0,1)$, $\alpha>0$, $U\subset\mathbf{R^n}(n\le 3)$ with a smooth boundary ∂U, $u=u(t,x)$ is a real-valued function on $U\times[\tau,+\infty)$, $\tau\in\mathbf{R}$, the functions f, g satisfy certain conditions which will be specified later. $W(t)$ is a two-side real-valued Wiener process on a probability space. The symbol "$\circ$" means that the stochastic equation is understood in the sense of Stratonovich's integration. When $s\in(0,1)$, the operator $(-\Delta)^s$ is so-called fractional Laplacian, which includes the integral and spectral fractional operators on bounded domain U, and they have different eigenfunctions and eigenvalues [1]. When $s=1$, $(-\Delta)^s$ becomes the standard Laplace operator $-\Delta$.

In recent years, fractional partial differential equations have appeared in a wide range of fields, within in physics, biology, chemistry, financial mathematics, etc., and some classical partial differential equations with fractional derivative have also been extensively studied, including fractional Schr*ö*dinger equations [2, 3], fractional Ginzburg-Landau equations [4–9], fractional Landau-Lifshitz equations [10], fractional Landau-Lifshitz-Maxwell equations [11] and fractional reaction-diffusion equations [12].

Stochastic partial differential equations(SPDEs) arise naturally in a wide variety of applications with uncertainties or random influences, called noises, are considered. During the past decades, analysis of infinite dimensional random dynamical system(RDS) has become an important field in the study of qualitative behaviour of SPDEs. The concept of pullback random attractor, which is a generalization of global attractor in deterministic systems (see [13–16]), was introduced in [17–22], and characterizes the long-time behavior of RDS perfectly. The existence of random attractors for SPDEs have been widely discussed by many authors, see, e.g., [23–31] in the autonomous SPDEs, and [12, 32–46] in the non-autonomous case.

As far as we are concerned, the finite dimensionality of an attractor is an important topic in describing the asymptotic behavior of infinite-dimensional dynamical systems. Up to now, there have several useful methods to estimate the upper bound of the Hausdorff and fractal dimensions of a random attractor. There is a fact that if a compact set A in a metric space has a bounded fractal dimension $\dim f(A)<m/2$ for some $m\in N$, then A can be placed in the graph of a Hölder continuous mapping which maps a compact subset of $\mathbf{R^m}$ onto A. This indicates that the finiteness of fractal dimension of an attractor plays a very important role in the finite-dimensional reduction theory of an infinite dimensional system. However, just knowing the boundedness of Hausdorff dimension of an attractor for a system, we still have no available finite parameterization. Recently, Zhou et al. discussed the fractal dimension of random attractors of non-autonomous stochastic reaction-diffusion equation in [47], Wang discussed the asymptotic behavior of