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Global Existence of Smooth Solutions to Three Dimensional Hall-MHD System with Mixed Partial Viscosity

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Abstract. We investigate the global existence of smooth solutions to the three dimensional generalized Hall-MHD system with mixed partial viscosity in this work. The diffusion of mixed partial viscosity is weaker than that of full viscosity, which cases new difficulty in proving global smooth solutions. Moreover, Hall term heightens the level of nonlinearity of the standard MHD system. Global smooth solutions are established by using energy method and the bootstrapping argument, provided that the initial data is enough small.

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Key Words: Hall-MHD system with mixed viscosity; global existence; smooth solutions.

1 Introduction

In this work, we consider smooth solutions to the following Cauchy problem of three dimensional Hall-MHD system with mixed partial viscosity

$$\begin{cases} \partial_t u+(u\cdot\nabla)u-\sum_{i=1}^{3}\mu_i\partial_{x_i}^2 u+\nabla p=(H\cdot\nabla)H, \\ \partial_t H+(u\cdot\nabla)H-\sum_{i=1}^{3}\nu_i\partial_{x_i}^2 H=(H\cdot\nabla)u-\nabla\times((\nabla\times H)\times H), \\ \nabla\cdot u=0, \qquad \nabla\cdot H=0, \end{cases} \tag{1.1}$$

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with the initial value

$$u(x,0)=u_0(x), \quad H(x,0)=H_0(x). \tag{1.2}$$

Here $u=u(x,t)$, $H=H(x,t)\in\mathbb{R}^3$, $p=p(x,t)\in\mathbb{R}$ are the velocity, magnetic and pressure field, respectively, and $\mu_i\geq 0$, $\nu_i\geq 0$ represent the kinematic viscosity and diffusivity constants, respectively.

As the incompressible limit of a two-fluid isothermal Euler-Maxwell system for electrons and ions, the Hall-MHD system (1.1) was derived in [1]. It describes the evolution of a system consisting of charged particles that can be approximated as a conducting fluid, in the presence of a magnetic field H, with u denoting the fluid velocity, p the pressure, μ_i the viscosity, ν_i the magnetic resistivity and η a constant determined by the ion inertial length. The Hall-MHD system has a wide range of applications in plasma physics and astrophysics, including modelling solar wind turbulence, designing tokamaks as well as studying the origin and dynamics of the terrestrial magnetosphere. Moreover, the Hall-MHD system also serves a vital role in interpreting the magnetic reconnection phenomenon, frequently observed in space plasmas. For more physical backgrounds, we may refer to [2–5] and [6].

The Hall-MHD system were mathematically rigorous derived by Acheritogaray, Degond, Frouvelle and Liu [1]. Existence of global solutions is a challenge open problem in the mathematical fluid mechanics. There are numerous important progresses on the fundamental issue of blow up criterion of smooth solutions or regularity criterion of weak solutions to (1.1), (1.2) (see [7–13] and [14]). Blow up criterion and global small solutions have been established in Chae and Lee [15]. Chae [16] proved that existence of global weak solutions and local classical solutions. Time-decay rate of solution was established in [17]. A stability theorem for global large solutions under a suitable integrable hypothesis in which one of the parcels is linked to the Hall term was proved in [18]. As a byproduct, a class of global strong solutions was also obtained with large velocities and small initial magnetic fields. Global well-posedness of mild solutions in Lei-Lin function spaces (see [19]) was established in [20]. Global well-posedness and analyticity of mild solutions was obtained by Duan [21]. By exploring the nonlinear structure, Zhang [22] constructed a class of large initial data and proved global existence of smooth solutions. Fan et al. [23] established global axisymmetric solutions. Wan and Zhou [24] proved that global existence and large time behavior of strong solutions. Chae and Weng [25] studied singularity formation for the incompressible Hall-MHD system without resistivity. For other some results, we refer to [26].

If $\nabla\times((\nabla\times H)\times H)$ disappear, (1.1) is reduced to the classical MHD system. For our purpose, we emphasize on the global smooth solutions to MHD system and related models with mixed viscosity, see [27–32]. Cao and Wu [27] proved that global regularity for the 2D MHD system with mixed partial dissipation and magnetic diffusion, provided that the initial data belongs to H^2. Wang and Wang [29] overcome these difficulties caused by more bad terms and extended the results to the 3D case with mixed