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The Averaging Principle for Stochastic Fractional Partial Differential Equations with Fractional Noises

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Abstract. The purpose of this paper is to establish an averaging principle for stochastic fractional partial differential equation of order $\alpha > 1$ driven by a fractional noise. We prove the existence and uniqueness of the global mild solution for the considered equation by the fixed point principle. The solutions for SPDEs with fractional noises can be approximated by the solution for the averaged stochastic systems in the sense of p-moment under some suitable assumptions.

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1 Introduction

Fractional calculus has attracted many physicists, mathematicians and engineers due to the contributions which have been made to both theory and applications of fractional (partial) differential equations(see, e.g., [1] and references therein). Mueller [2] and Wu [3] proved the existence of a solution of the stochastic fractional heat equation. Then, Bonaccorsi and Tubaro [4] applied Mittag-Leffler's function to explore stochastic evolution equations with fractional time derivatives. After that, Cui and Yan [5] studied the existence of mild solutions for a class of fractional neutral stochastic integro-differential equations with infinite delay in Hilbert spaces; In [6], Liu and Yan have established the existence and uniqueness of solutions to a jump-type stochastic fractional partial differential equation with fractional noises by fixed point theorem.

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The averaging principle plays a crucial role to obtain the approximation solutions for differential equations dating from mechanics, molecular dynamics, mathematics, material science, and other areas of sciences and engineering. Some rigorous results on the approximation theorem to the solutions of stochastic differential equations can be dated back to Khasminskii [7,8]. Based on this work, recently, Xu et al. [9] have established the averaging principle for the solutions of stochastic partial differential equations driven by Lévy noise under Lipschitz and linear growth conditions. Peculiarly, they have proved that the solutions to the simplified systems converge to that of the corresponding original systems both in the sense of mean square and probability. Similar results were proposed to the multivalued stochastic differential equations by [10]. Not only that, Pei et al. [11] established the averaging principles for a class of stochastic partial differential equations with slow component driven by fractional Brownian motion and a fast one driven by a fast-varying diffusion. In case with Poisson random measure was studied in Pei et al. [12] and α-noise in Bao et al. [13]. An averaging principle for the heat equation driven by a general stochastic measure was studied by Radchenko [14].

On the other hand, there has been some recent interest in studying stochastic partial differential equations driven by a fractional noise. For example, Duncan et al. [15] considered linear stochastic evolution equations in a Hilbert space driven by an additive cylindrical fractional Brownian motion with $H\in(\frac{1}{2},1)$ and Tindel et al. [16] provided necessary and sufficient conditions for the existence and uniqueness of an evolution solution. Many interesting works on stochastic partial differential equations driven by fBm have been done and we refer to the literatures [17–19].

Based on the above brief discussion and to the author's best knowledge, the averaging principle for stochastic fractional partial differential equations with fractional noises has not been considered. Therefore, in this paper, we will consider this issue by studying the following stochastic fractional partial differential equation with fractional noises:

$$\begin{cases}\dfrac{\partial u}{\partial t}=D^{\alpha}_{\delta}u+f(t,x,u)+\sigma(t,x)\dot{B}^H, & \text{in } [0,T]\times\mathbb{R},\\ u(0,\cdot)=u_0(\cdot),\end{cases} \tag{1.1}$$

where D^{α}_{δ} is the fractional differential operator with respect to the spatial variable, to be defined in the Appendix which was recently introduced by Debbi [20] and Debbi and Dozzi [21], $\dot{B}^H$ denotes the fractional noise on $[0,T]\times\mathbb{R}$ with Hurst index $H>\frac{1}{2}$ defined on a complete probability space $(\Omega,\mathcal{F},P)$ (see Section 2 for precise definitions). In fact, we understand this equation in the Walsh [22] sense, and so we can rewrite Eq. (1.1) as follows:

$$\begin{aligned}u(t,x)=&\int_{\mathbb{R}}G_\alpha(t,x-y)u_0(y)\mathrm{d}y+\int_0^t\int_{\mathbb{R}}G_\alpha(t-s,x-y)\sigma(s,y)B^H(\mathrm{d}s,\mathrm{d}y)\\&+\int_0^t\int_{\mathbb{R}}G_\alpha(t-s,x-y)f(s,y,u(s,y))\mathrm{d}y\mathrm{d}s,\end{aligned} \tag{1.2}$$