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The Nonexistence of the Solutions for the Non-Newtonian Filtration Equation with Absorption

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Abstract. The paper proves the nonexistence of the solution for the following Cauchy problem

$$\begin{cases} u_t = \operatorname{div}\left(|\nabla u^m|^{p-2}\nabla u^m\right) - \lambda u^q, & (x,t)\in S_T = \mathbb{R}^N\times(0,T),\\ u(x,0)=\delta(x), & x\in\mathbb{R}^N, \end{cases}$$

under some conditions on m,p,q,λ, where δ is Dirac function.

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1 Introduction

The polytropic filtration equation $u_t = \operatorname{div}\left(|\nabla u^m|^{p-2}\nabla u^m\right)$ has profound physical background [1]. If p=2, it is a porous media equation. If m=1, it is an evolutionary p-Laplacian equantion. In this paper, we mainly pay attention on the following Cauchy problem in which the initial value is a Dirac measure

$$u_t = \operatorname{div}\left(|\nabla u^m|^{p-2}\nabla u^m\right) - \lambda u^q, \tag{1.1}$$

$$u(x,0) = \delta(x). \tag{1.2}$$

We denote $B_R = \left\{x\in\mathbb{R}^N : |x| < R\right\}$, and introduce the following definition.

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Definition 1.1. *A nonnegative function $u(x,t)$ is said to be a weak solution of the Cauchy problem* (1.1)-(1.2)*, if for any $\tau>0$, $R>0$,*

$$u\in C\Big([0,T];L^1\Big(\mathbb{R}^N\Big)\Big)\bigcap L^\infty\Big(\mathbb{R}^N\times(\tau,T)\Big), \tag{1.3}$$

$$u^m\in L^p_{loc}\Big([0,T];W^{1,p}(B_R)\Big), \tag{1.4}$$

and for any function $\varphi\in C^1_0(S_T)$, $\chi\in C^1_0(\mathbb{R}^N)$, $u(x,t)$ satisfies

$$\iint_{S_T}\Big[u\varphi_t-|\nabla u^m|^{p-2}\nabla u^m\nabla\varphi-\lambda u^q\varphi\Big]\mathrm{d}x\mathrm{d}t=0, \tag{1.5}$$

and

$$\lim_{t\to 0}\int_{\mathbb{R}^N}\chi(x)u(x,t)\mathrm{d}x=\chi(0). \tag{1.6}$$

Gmira proved the nonexistence of the solutions of Cauchy problem (1.1)-(1.2), when λ=1, p>2, m>0, q>0, $q\geq m(p\text{-}1)+\frac{p}{N}$ [2]. Zhan proved there is not nonnegative solution for Cauchy problem (1.1)-(1.2), when λ=1, 1< p<2, m>1, q> p-1, and $2\text{-}p+\frac{p}{N}<m(p\text{-}1)+\frac{p}{N}\leq 1$ [3]. Chen showed the Cauchy problem (1.1)-(1.2) admits a weak solution in the sense of Definition 1.1 with λ=1, p>2, m>0, q>0, $m(p\text{-}1)<q<m(p\text{-}1)+\frac{p}{N}$ [4]. Yang and Zhao have researched the existence of the solutions of the problem (1.1)-(1.2) for m=1 [5,6]. In this paper, after complicate calculations, the results in [5] are extended to the following general results.

Theorem 1.1. *Suppose that*

$$1<p\leq\frac{(m+1)N}{mN+1},\ \ \lambda\geq 0,\ \ q\geq 0,\ \ \max\left(\frac{1}{p(p-1)},\frac{2-p}{p-1}\right)<m<\frac{1}{p-1}. \tag{1.7}$$

Then Cauchy problem (1.1)-(1.2) *has no solution.*

Zhao and Yuan have verified the existence and uniqueness of the solution of Cauchy problem (1.1)-(1.2) when $\lambda=0, m>\frac{1}{p-1}$ and $u_0(x)\in L^1(\mathbb{R}^N)$ [7].

2 Proof of Theorem 1.1.

Lemma 2.1. *Suppose that*

$$1<p<2,\ \lambda\geq 0,\ q\geq 0,\ \max\left(\frac{1}{p(p-1)},\frac{2-p}{p-1}\right)<m<\frac{1}{p-1}. \tag{2.1}$$

Then the solution $u(x,t)$ of the Cauchy problem (1.1)-(1.2) *satisfies*

$$\sup_{0<\tau<t}\int_{B_R}u(x,\tau)\mathrm{d}x\mathrm{d}\tau\leq c+ct^{\frac{1}{1-mp+m}}R^{N-\frac{p}{1-mp+m}}, \tag{2.2}$$