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Integrated Pest Management System with Impulsive Control of Spatial Heterogeneity

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Abstract. An impulsive integrated pest management system with diffusion is investigated within this paper. The conditions for pest eradication of the impulsive system without natural enemies are established based on the Krein-Rutman theorem and the comparison principle for parabolic equations. Integrated pest management can be achieved at an exponential rate, when the principal eigenvalues of the auxiliary system is large enough. Numerical simulations are presented to demonstrate the theoretical results. A discussion is given at the end.

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Key Words: Integrated pest management; eigenvalue problem; pest control; diffusion.

1 Introduction

The concept of integrated pest management (IPM) was introduced in the late 1950s and was widely practised during the 1970s and 1980s [1–4]. It is defined as a process consisting of the balanced use of all the pest controls that are environmentally compatible, economically feasible, and socially acceptable to reduce pest populations to tolerable levels. And pest control often involves biological, cultural, and chemical control [1–6].

Biological control, which is defined as the suppression or the elimination of pest populations by natural enemies, has been an important aspect in an IPM strategy [6]. Natural

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enemies used for biological control can be divided into three categories: Predatory enemies, including lacewings, ladybugs and many insectivorous birds; Parasitic enemies, including parasitic wasps, parasitic flies, etc; The pathogenic microorganism, including the bacterium, bacillus thuringiensis, and so on. This control strategy is usually used by releasing natural enemies at a critical time to reduce a pest's population [5,7,8]. Biological control has been applied to greenhouse culture, the use of Encarsia formosa against Trialeurodes vaporariorum on tomatoes and cucumbers is one of the first successful cases of biological control in greenhouses [3,4,9]. Another important method for pest control is chemical control. In most cropping systems, pesticide are still the principal means of controlling pests. They can be cheap and are easy to apply, act fast [5], but chemical control also has certain harmfulness, pesticide can not only environmental pollution, but also cause plant phytotoxicity and waste of resources, and if incorrectly used, also easily lead to human and animal poisoning.

In mid 80s, there has been renewed interest in modelling IPM. Many IPM strategies such as releases of natural enemies at critical times and killing pests instantly by spraying pesticides have been proposed by mathematical models [6–16]. In [7], Liang et al. developed two novel pest-natural enemy interaction models incorporating the evolution of pesticide resistance, they investigated the number of natural enemies to be released when threshold conditions for the extinction of the pest population in two different control tactics are reached. In [16], Tang et al. modelled IPM including residual effects of pesticides in terms of fixed pulse-type actions.

However, the aforementioned above studies ignore the effect of space. The questions that arise are: in the case of uneven distribution of space, if we aim to eradicate the pest, how do we release the natural enemies? what proportion do we need to kill the pests by pesticide? To address these questions, we present an impulsive differential equation with diffusion. In the established model, (a) insect and natural enemies are all dealt with control in space; (b) distributed control of pest are considered by a combination of the action of natural enemies and impulsive control which include pesticide spraying and natural enemies releasing; (c) because of the pesticide's impact on the natural enemies, a proportion of the natural enemies could be killed at the time the pesticide is sprayed to kill the pests [17–19]. When we ignore the effect of space, the established model can be found in many domain of the applied sciences [20–22].

The aim of this work is to determine the conditions for pest eradication by pesticide spray-ing at critical times and the role of natural enemies releasing in enhancing the control. In order to show the conditions of pest extinction, Firstly, we discuss the model without natural enemies and get the sufficient and necessary conditions of pest extinction. Secondly we analyze the model without pest and we find the positive periodic solution of natural enemies. By using the conclusion of Section 3 and Section 4, we finally get the conditions for the extinction of the pests. Examples and numerical simulations are presented in Section 6 to illustrate the feasibility of our results. Section 7 involves some concluding remarks and discussions.