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Stochastic Averaging Principle for Mixed Stochastic Differential Equations

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Abstract. In this paper, an averaging principle for the solutions to mixed stochastic differential equation involving standard Brownian motion, a fractional Brownian motion B^H with the Hurst parameter $H > \frac{1}{2}$ and a discontinuous drift was estimated. Under some proper assumptions, we proved that the solutions of the simplified systems can be approximated to that of the original systems in the sense of mean square by the method of the pathwise approach and the Itô stochastic calculus.

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Key Words: Averaging principle; mixed stochastic differential equation; discontinuous drift; fractional Brownian motion.

1 Introduction

In this paper, we consider the following mixed stochastic differential equation:

$$X(t)=X_0+\int_0^t a(s,X(s))ds+\int_0^t b(s,X(s))dW(s)+\int_0^t c(s,X(s))dB^H(s), \tag{1.1}$$

where $X_0 \in \mathbb{R}$, $t \in [0,T]$, $a,b,c:[0,T]\times\mathbb{R}\to\mathbb{R}$, and B^H is a fractional Brownian motion with the Hurst parameter $H \in (\frac{1}{2},1)$. The processes W and B^H can be dependent. The integral w.r.t. Wiener process W is a standard Itô integral, and the integral w.r.t. B^H is the pathwise generalized Lebesgue-Stieltjes integral which is defined in [1, 2, 3].

The strongest motivation to study such equations comes, in particular, from financial modeling. It is well known that there are two kinds of sources that affect prices in

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financial markets. The random noise from the first source usually prevails in a shorter time periods and can be modeled by a Wiener process W. While the random noise of the second source is coming from economical background and usually has a long range dependence property, which can be modeled by a Hölder continuous process with exponent greater than $\gamma > \frac{1}{2}$, just like fractional Brownian motion. Consequently, the mixed model describes the stock price behavior in a better way. Moreover, one can note that the long-range influence in the financial markets and application of the models involving fractional Brownian motion and Brownian motion were established in many papers (see [4, 5, 6, 7, 8] and references therein).

Recently, the mixed stochastic differential equations (1.1) have attracted a great deal of attention. For example, Guerra and Nualart and Kubilius considered the existence and uniqueness for the strong solution to Eq. (1.1) in [3] and [9]; Mishura and Shevchenko[10] studied the the existence, uniqueness and convergence of the solution to Eq. (1.1); and in [11], Mishura et al. investigated the the convergence of Euler approximations for mixed delay stochastic differential equations; while Liu and Luo discussed the mean square rate of convergence by using a modified Euler method in [12]; Melnikov et al. [13] obtained the stochastic viability and comparison theorems to Eq. (1.1). We notice that the shift coefficients a in the above papers are continuous in X. But, in many practical applications, the continuity of the shift coefficients is too strong to be satisfied. The first motivation of our work is to obtain the existence of strong solution to Eq. (1.1) when a can be discontinuous in X.

Stochastic averaging principle, which enables to study the complex equations in terms of the related averaged one, plays an important role in multiscale problem (see, e.g. [14,15,16,17,18,19] and references therein). A stochastic averaging technique for SDEs only driven by fBm with Hurst parameter $H \in (\frac{1}{2},1)$ was developed by Xu et al. [20,21], and they proved that the original system can be approximated by the averaged systems in the method of mean square convergence and of convergence in probability; Li and Yan [22] estabilished the stochastic averaging for two-time-scale stochastic partial differential equations driven by fractional Brownian motion with Hurst parameter $H \in (0,\frac{1}{2})$. After that, Pei et al.[23] studied the averaging principle for multidimensional, time dependent, Eq. (1.1) with the situation that the drift term is continuous by combining the pathwise approach with the Itô stochastic calculus. Motivated by this work, the second motivation of our work is to consider a following interesting question: when $\gamma > \frac{1}{2}$ in Eq. (1.1), can the stochastic averaging principle establish if the coefficient a is discontinuous? More precisely, we will establish the stochastic averaging principle for the strong solution of Eq. (1.1) when $a(t,x)$ has linear growth and is left-continuous and lower semi-continuous in x.

The rest of this paper proceeds as follows. Section 2 presents preliminary results that are needed in the subsequent section. In Section 3, the existence of the strong solution to Eq. (1.1) will be established. A stochastic averaging principle for SDEs driven by standard Bm and fractional Brownian motion with the Hurst parameter $H > \frac{1}{2}$ will be