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GLOBAL EXISTENCE OF SUPERSONIC FLOW PAST A CURVED CONVEX WEDGE *

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Dedicated to Professor Ding Xiaxi on the occasion of his 70th birthday

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Abstract In this paper we discuss the supersonic flow past a curved convex wedge. Our conclusion is that if the vertex angle of the wedge is less than a critical angle, the shock attached the head of the wedge is weak, and if the wedge is formed by a smooth convex curve, monotonically increasing, then the global solution of such a boundary value problem exists.

Key Words Global existence; supersonic flow; quasilinear hyperbolic system.
Classification 35L60, 35L67, 35L50, 76N15.

1. Introduction

It is well known that when a supersonic flow hits a wedge with small vertex angle, there will appear an oblique shock attached on the edge of the wedge. If the surface of the wedge is a smooth curved surface, then by its influence the shock front will also be a curved surface. It is shown in [1] that if the wedge has constant section and the vertex angle is less than a critical value, then the shock front and the flow behind the shock can be determined locally. An interesting and important question is then whether we can determine the flow with the shock front globally? When the surface of the wedge is composed of two straight lines combined by a smooth curve, the global solution can be obtained by constructing infinite reflection of rarefaction waves (See [2]). In this paper we will use a different way to discuss such a problem. Our conclusion is that if the shock is weak, the vertex angle of the wedge is less than the critical angle, and if the surface of the wedge is formed by a smooth convex curve, monotonically increasing, then the global solution does exist.

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2. Description of the Problem and Its Reduction

Let us first give a precise description of the problem. Assume that the wedge is symmetric with respect to a centre plane, then we always consider the upper half of the wedge. Assume also that the surface of the wedge has constant section on any plane perpendicular to its edge, and the equation of the surface is $y = f(x)$ satisfying $f'(x) > 0$, $f''(x) \le 0$. Besides, the flow is assumed to be isentropic and irrotational, this assumption is acceptable if the possible shock is weak. In this case the system to describe the flow can be written as

$$\begin{cases} (u^2 - a^2)u_x + uv(u_y + v_x) + (v^2 - a^2)v_y = 0 \\ u_y = v_x \end{cases} \tag{1}$$

where a represents the sonic speed. These unknown functions also satisfy Bernoulli relation

$$\frac{1}{2}(u^2 + v^2) + \frac{a^2}{\gamma - 1} = const \tag{2}$$

Ahead of the shock the flow is constant with its parameters $u = u_0$, $v = v_0$, $\rho = \rho_0$, satisfying $u_0 > a_0$, $v_0 = 0$. So the constant in (2) equals $\frac{1}{2}u_0^2 + \frac{1}{\gamma - 1}a_0^2$.

Denote the location of the unknown shock by $y = s(x)$, we consider a boundary value problem of (1) in the domain

$$\Omega : x > 0, \quad f(x) \le y \le s(x) \tag{3}$$

while its boundary is denoted by $B : y = f(x)$ and $S : y = g(x)$. And the boundary conditions are

$$v = uf'(x) \quad \text{on } B \tag{4}$$

$$u + vs'(x) = u_0, \quad \rho(us'(x) - v) = \rho_0 u_0 s'(x) \quad \text{on } S \tag{5}$$

where the condition (5) is called Rankine-Hugoniot condition.

The system can be diagonalized by introducing Riemann invariants. Denote

$$\lambda_{\pm} = \frac{uv \pm a\sqrt{u^2 + v^2 - a^2}}{u^2 - a^2} \tag{6}$$

which represents the characteristic directions of the system (1). Then by introducing suitable integral factor $k_{\pm}$ we can define functions r and s by

$$\begin{cases} ds = k_-(du + \lambda_- dv) \\ dr = k_+(du + \lambda_+ dv) \end{cases} \tag{7}$$