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ASYMPTOTIC BEHAVIOUR OF BLOW-UP FOR SOLUTIONS OF SEMILINEAR REACTION-DIFFUSIONAL SYSTEMS

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Abstract In this paper, we consider the asymptotic behaviour of the solutions of semilinear reaction-diffusional systems and obtain the growth rate.
Key Words Blow-up point; reaction-diffusional system; similar variables.
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1. Introduction

In this paper we will consider the asymptotic behavior of the symmetric solutions of semilinear reaction-diffusion systems:

$$u_t = u_{xx} + \lambda e^v, \quad (x,t) \in S_T = [-l,l] \times (0,T) \tag{1.1a}$$

$$v_t = v_{xx} + \mu e^u, \quad (x,t) \in S_T = [-l,l] \times (0,T) \tag{1.1b}$$

in a neighborhood of the blow–up point as t approaches the finite blow-up time $T < \infty$, where λ, μ are positive constants and $l < \infty$ (Dirichlet problem) or $l = \infty$ (Cauchy problem). Branell, Lacay and Wake [1] and Pao [2] have discussed the system (1.1) with blow-up, the solution $(u(x,t), v(x,t))$ blows up in finite time $T < \infty$ i.e. $\limsup_{t\to T} u(x,t) = \limsup_{t\to T} v(x,t) = \infty$. On the other hand, a point $x \in [-l,l]$ is called a blow-up point of $u(x,t)$ if there is a sequence (x_n, t_n) such that $t_n \uparrow T$, $x_n \to x$, and $u(x_n, t_n) \to \infty$ as $n \to \infty$ where T is the blow-up time. Friedman and Giga [3] described the single blow-up point for parabolic system (1.1) under certain conditions.

For the solid fuel ignition model

$$u_t = \Delta u + e^u, \quad x \in B_r \subset R^n \quad \text{and} \quad 0 < t < T \tag{1.2}$$

Beberwes, Bressan and Eberly [4] characterized the asymptotic behavior of the solution $u(x,t)$ of (1.2) near a blow-up singularity, provided $n = 1, 2$ and established that the solution $u(x,t)$ of (1.2) with suitable initial and boundary conditions satisfies

$$u(x,t) + \ln(T-t) \to 0$$

uniformly on $0 \le |x| \le c(T-t)^{1/2}$ $c > 0$ as $t \to T^-$.

The main result of this paper is Theorems 4.7 and 5.1 which give the asymptotic behaviours where the solution $u(x,t)$ and $v(x,t)$ satisfy

$$u(x,t) + \ln(T-t) + \ln\mu \to 0 \quad \text{and} \quad v(x,t) + \ln(T-t) + \ln\lambda \to 0$$

uniformly on $0 \le |x| \le c(T-t)^{1/2}$ $c > 0$ as $t \to T^-$.

Here, we make use of the methods of [4, 5]. In Section 2, we prove Lemma 2.1 which shows the behavior near blow-up point. In Section 3, we define the similar change of variables and study self-similar solution. In Section 4, the result for Dirichlet initial-boundary problem will be obtained. In Section 5, we obtain the result for Cauchy problem.

2. Preliminaries

Consider the system

$$u_t = u_{xx} + \lambda e^v, \quad (x,t) \in S_T \tag{2.1a}$$
$$v_t = v_{xx} + \mu e^u, \quad (x,t) \in S_T \tag{2.1b}$$

with initial and boundary value

$$u(\pm l, t) = 0 \quad \text{and} \quad v(\pm l, t) = 0 \quad \text{for} \quad 0 < t < T \tag{2.2a}$$
$$u(x,0) = u_0(x) \quad \text{and} \quad v(x,0) = v_0(x) \quad \text{for} \quad -l < x < l \tag{2.2b}$$

where $\lambda > 0$, $\mu > 0$, and assume that

$$\left.\begin{array}{l} u_0(x), v_0(x) \text{ are symmetric respect to the origin and continuous,} \\ \text{nonnegative and nonincreasing in } [0,1] \text{ and } u_0(\pm l) = v_0(\pm l) = 0 \\ u_0'' + e^{v_0} \ge 0 \quad \text{and} \quad v_0'' + e^{u_0} \ge 0 \end{array}\right\} \tag{2.3}$$

Under the assumption (2.3), Friedman and Giga [3] have proved that blow-up occurs only at the origin $x = 0$ for $u(x,t)$ and $v(x,t)$ in the same time. By the maximum