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ON REGULARITY FOR SOLUTION OF NON-LINEAR EQUATION WITH CONSTANT MULTIPLE CHARACTERISTIC*

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Abstract In this paper we consider the propagation of microlocal regularity near constant multiple characteristic of a real solution $u \in H^s$ $(s > m + \max\{\mu, 2\} + \frac{n}{2})$ of non-linear partial differential equation

$$F(x, u, \cdots, \partial^\beta u, \cdots)_{(|\beta| \le m)} = 0$$

We will prove that the microlocal regularity near constant multiple characteristic of the solution u will propagate along bicharacteristic with constant multiplicity μ and have loss of smoothness up to order $\mu - 1$ under Levi condition.

Key Words Constant multiple characteristic; Levi condition; paradifferential operator; bicharacteristic; para-linearization.
Classification 35L35, 35S05.

1. Introduction

Let us consider a fully non-linear partial differential equation

$$F(x, u, \cdots, \partial^\beta u, \cdots)_{(|\beta| \le m)} = 0 \tag{1.1}$$

and a solution u belonging to $H^s_{loc}(\Omega)$, $s > m + \frac{n}{2} + \max\{\mu, 2\}$, $\Omega \subset \mathbf{R}^n$, which is an open set, and F is a C^∞-function near $(x_0, y_0^0, \cdots, y_\beta^0, \cdots)$, where $y_\beta^0 = \partial^\beta u(x_0)$, $|\beta| \le m + \max\{\mu, 2\}$. Let $\widetilde{F}(x, \partial u(x), D_x)$ be the para-linearization operator of non-linear equation (1.1)

$$\widetilde{F}(x, \partial u(x), D_x) v(x) = \sum_{|\alpha| \le m} \frac{\partial F}{\partial y_\alpha}(x, \cdots, \partial^\beta u(x), \cdots) \partial^\alpha v(x) \tag{1.2}$$

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If (x_0, ξ^0) is a simple characteristic point of $\mathrm{Char}(\widetilde{F}(x, \partial u(x), D_x))$, so $\widetilde{F}(x, \partial u(x), D_x)$ is of principal type near (x_0, ξ^0), then microlocal regularity of u propagates along bicharacteristic issued from (x_0, ξ^0). In case of lower frequency, the theorem is proved by J.M. Bony [1]. J.M. Bony [2, 3], S. Alinhac [4] and M. Beals and G. Métivier [5, 6] also studied the propagation, interaction and reflection of waves with conormal singularity under strictly hyperbolic condition. What if $\widetilde{F}(x, \partial u(x), D_x)$ is not principal type at (x_0, ξ^0)? In this case there are three problems: the first is how to symmetrize the para-linearization operator $\widetilde{F}(x, \partial u(x), D_x)$, the second problem is how to microlocalize the symmetric para-differential operator, namely how to construct a class of microlocal operators? If these problems are solved, then we can analyse the microlocal regularity of solution u of non-linear PDE. The third problem is, in case of multiple characteristic, how to work with similar conormal singularity. The author considered singularity of the solution of a class non-linear partial differential equation with involutive multiple characteristics [7] and constant multiple characteristics [8]. When the multiplicity of characteristics of $\widetilde{F}(x, \partial u(x), D_x)$ for solution u is constant, under a proper condition (namely Levi condition), the operator $\widetilde{F}(x, \partial u(x), D_x)$ can be reduced to a system with diagonal principal part, where operators on diagonal are of principal type. Then we can construct a class of microlocal operators as in Bony [1], and the characteristic hypersurfaces of diagonal operators are also characteristic hypersurfaces of the operator $\widetilde{F}(x, \partial u(x), D_x)$. Then we can also consider conormal singularities. So the key problem is how to give the Levi condition. We know that there are many results on microlocal analysis of linear partial differential operators with multiple characteristics under some suitable Levi condition. Particularly, P.J. Chazarain made a log of researches on microlocal analysis of linear partial differential operators with constant multiple characteristics and he gave the Levi condition represented by characteristic function [9].

In this paper we consider the propagation of microlocal regularity of solution for a class of fully non-linear partial differential equation with constant multiple characteristic. Suppose that the Levi condition, which is the extension of the Levi condition in P.J. Chazarain [9], is satisfied, then we will prove that the microlocal regularity of solution propagating along bicharacteristic with constant multiplicity μ will endure a loss of regularity of degree $\mu - 1$. Then we will consider a quasi-linear equation

$$\sum_{|\alpha|=m} A_\alpha(x, \cdots, \partial^\beta u, \cdots)\partial^\alpha u(x) = B(x, \cdots, \partial^\beta u, \cdots), \quad |\beta| < m \tag{1.3}$$

and obtain a similar result under weaker hypotheses on parameter s, t.