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THE GLOBAL SMOOTH SOLUTION FOR LANDAU-LIFSHITZ-MAXWELL EQUATION WITHOUT DISSIPATION

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Abstract In this paper, the global existence of a unique smooth solution for the Landau-Lifshitz-Maxwell equations of the ferromagnetic spin chain without dissipation in n $(1 \le n \le 2)$ dimensions is established by using the viscosity elimination method.

Key Words Landau-Lifshitz-Maxwell equation; global smooth solution.
Classification 35K57, 35B40.

1. Introduction

In 1935, Laudau-Lifshitz [1] proposed the following coupled system of the nonlinear evolution equation

$$\vec{z}_t = \lambda_1 \vec{z} \times (\Delta \vec{z} + \vec{H}) - \lambda_2 \vec{z} \times (\vec{z} \times (\Delta \vec{z} + \vec{H})) \tag{1.1}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} \tag{1.2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{H}}{\partial t} - \beta \frac{\partial \vec{z}}{\partial t} \tag{1.3}$$

$$\nabla \cdot \vec{H} + \beta \nabla \cdot \vec{z} = 0, \quad \nabla \cdot \vec{E} = 0 \tag{1.4}$$

where $\lambda_1, \lambda_2, \sigma, \beta$ are constants, $\lambda_2 \ge 0$, $\sigma \ge 0$, $\vec{z}(x,t) = (z_1(x,t), z_2(x,t), z_3(x,t))$ denotes the microscopic magnetization field, $\vec{H} = (H_1(x,t), H_2(x,t), H_3(x,t))$ the magnetic field, $\vec{E}(x,t) = (E_1(x,t), E_2(x,t), E_3(x,t))$ the electric field, $\vec{H}^e = \Delta \vec{z} + \vec{H}$ the effective magnetic field, $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$, $\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \cdots, \frac{\partial}{\partial x_n}\right)$, "×" the cross product of the vector in $\mathbf{R}^3$.

If $\vec{H}=0$, $\vec{E}=0$, we obtain the Landau-Lifshitz system with Gilbert term

$$\vec{z}_t=\lambda_1\vec{z}\times\Delta\vec{z}-\lambda_2\vec{z}\times(\vec{z}\times\Delta\vec{z}) \tag{1.5}$$

where $\lambda_2>0$ is a Gilbert damping coefficient. In [2–4], the properties of the solution for the system of the equation (1.5) and the closed new links between the solution and the harmonic map on the compact Riemann manifold have been studied extensively. When $\lambda_2=0$, the system of the equation (1.5) becomes

$$\vec{z}_t=\lambda_1\vec{z}\times\Delta\vec{z} \tag{1.6}$$

In the case of $n=1$, it is an integrable system, and has soliton solutions. In [5–13], the authors have studied in detail the solitons for (1.6), the interaction among solitons, the infinite conservative laws, the inverse scattering method, and the relation with the nonlinear Schrödinger equations. As pointed out in [14], the system of the equation (1.6) is a strongly coupled degenerate quasilinear parabolic system. In [14–22], we have investigated extensively the classic and generalized solutions to the initial value problem and other kinds of boundary value problem for the system of the equation (1.6), and some properties of the solutions, and further obtained the global generalized solutions for $n\geq 2$. In [23] the existence and uniqueness of the global smooth solution for the periodic initial value problem and initial value problem of the system (1.1)–(1.4) for $n\leq 2$ are proved when $\lambda_2>0$.

In this paper we shall discuss the case of $\lambda_2=0$, i.e., the following Landau-Lifshitz-Maxwell system

$$\vec{z}_t=\lambda_1\vec{z}\times(\Delta\vec{z}+\vec{H}) \tag{1.7}$$

$$\nabla\times\vec{H}=\frac{\partial\vec{E}}{\partial t}+\sigma\vec{E} \tag{1.8}$$

$$\nabla\times\vec{E}=-\frac{\partial\vec{H}}{\partial t}-\beta\frac{\partial\vec{z}}{\partial t} \tag{1.9}$$

$$\nabla\cdot\vec{H}+\beta\nabla\cdot\vec{z}=0,\quad \nabla\cdot\vec{E}=0 \tag{1.10}$$

with the periodic initial value condition:

$$\vec{z}(x+2D,t)=\vec{z}(x,t),\quad \vec{H}(x+2D,t)=\vec{H}(x,t)$$
$$\vec{E}(x+2D,t)=\vec{E}(x,t),\quad x\in\mathbf{R},\ t\geq 0 \tag{1.11}$$

$$\vec{z}(x,0)=\vec{z}_0(x),\quad \vec{H}(x,0)=\vec{H}_0(x),\quad \vec{E}(x,0)=\vec{E}_0(x),\quad x\in\mathbf{R} \tag{1.12}$$