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SOME PERTURBATION PROBLEMS ON 2×2 NONLINEAR HYPERBOLIC CONSERVATION LAWS

Liu Hongxia
(Department of Mathematics, Jinan University, Guangzhou, 510632, China)
Lin Longwei
(Macao University, Macao)
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Abstract We study in this paper the perturbation of elementary waves with interactions: overtaking of shock waves belonging to the same characteristic family and penetrating of a shock wave and a rarefaction wave belonging to the different characteristic family for 2×2 genuinely nonlinear strictly hyperbolic conservation laws. The entropy solutions for the perturbed problems are obtained by the Glimm's scheme.

Key Words 2×2 genuinely nonlinear strictly hyperbolic conservation laws; perturbed problem; elementary waves with interaction.

Classification 35L.

1. Introduction

Consider the Cauchy problem to 2×2 nonlinear conservation laws

$$\begin{cases} U_t + F(U)_x = 0, \quad x \in \mathrm{R},\ t > 0 & (1.1) \\ U(x,0) = U_0(x), \quad x \in \mathrm{R} & (1.2) \end{cases}$$

where $U = (u,v)$, $U_0 = (u_0, v_0)$, $F = (f,g)$ is smooth in a region $P_0 \subset\subset \mathrm{R}^2$. We assume that the system (1.1) is strictly hyperbolic in the region P_0, i.e. the eigenvalues of the matrix dF are real and distinct, $\lambda_1 < \lambda_2$ in P_0. Let $r = r(U)$, $s = s(U)$ be the corresponding Riemann invariants. Moreover, the system (1.1) is genuinely nonlinear in Lax's sense, i.e. $\lambda_{1s}(U) > 0$, $\lambda_{2r}(U) > 0$ in P_0.

We study some perturbation problems for (1.1) (1.2). Suppose the initial data (1.1) are perturbed, the perturbed initial data are

$$\overline{U}(x,0) = \overline{U}_0(x), \quad x \in \mathrm{R} \tag{1.3}$$

where $\|\overline{U}_0(x) - U_0(x)\|$ is sufficiently small, $\|\cdot\|$ is a norm in a certain sense.

Definition 1 *The function* $\widetilde{U} \triangleq \overline{U}_0(x) - U_0(x)$ *is called perturbation of initial data* $U_0(x)$.

Definition 2 *The entropy solutions* $\overline{U}(x,t)$ *of the perturbed problem* (1.1) (1.3) *are called perturbed solutions of the corresponding unperturbed problem* (1.1) (1.2).

Let R_1 (resp. S_1) and R_2 (resp. S_2) be the rarefaction (resp. shock) waves corresponding to the first and second characteristic families respectively, then there are six nontrival two elementary wave interactions[1]:

(I) R_2 interacts with R_1;

(II) S_1 interacts with S_1 (or S_2 interacts with S_2);

(III) R_2 interacts with S_1 (or S_2 interacts with R_1);

(IV) S_2 interacts with S_1;

(V) S_2 interacts with R_2 (or R_1 interacts with S_1);

(VI) R_2 interacts with S_2 (or S_1 interacts with R_1).

In [2] [3], the authors solved the perturbation problem of elementary waves with interaction (I) for (1.1). In [4], we studied the perturbation of two elementary waves with interaction (II)–(VI) respectively and obtained the existence of the corresponding perturbed solution for the system of isentropic gas dynamics equations.

This paper is a continuation of [4], we study the perturbation of elementary waves with interactions (II) and (III) for general 2×2 system (1.1) under the condition (F) which will be given in Section 3. In other words, we study the existence of the perturbed solutions for (1.1) (1.2) under the condition (F), where the unperturbed initial data (1.2) are step functions and the entropy solutions for (1.1) (1.2) consist of rarefaction waves and discrete shock waves only with interaction (II) (III) (here, the unperturbed problems were solved by Smoller and Johnson[5]). The perturbed solutions are obtained by the Glimm's scheme.

2. Estimates on Interactions

To carry out our analysis, we improve the well-known framework given in [6] [1].

We now briefly describe the Glimm's difference scheme[6]. Choose mesh lengths l, h so that l/h is a constant, which is greater than $\max\{|\lambda_1|, |\lambda_2|\}$. In each step $nh < t < (n+1)h$ the approximate solution is exact and consists of elementary waves generated at the points of discontinuity $x = ml$, where $n \geq 0$, m are integers. At time $t = (n+1)h$, the value of the approximate solution $(u^l(x,t), v^l(x,t))$, $(m-1)l < x < (m+1)l$, is set to be the value of the exact solution in the strip $nh < t < (n+1)h$ at $t = (n+1)h$ and $x = (m + e_{n+1})l$. Here $\{e_n\}$ is a prechosen random sequence in $(-1, 1)$. The upper half (x,t)-plane is covered by diamonds $\Delta_{m,n}$ with vertices $a_{m,n-1}, a_{m+1,n}, a_{m,n+1}, a_{m-1,n}$,