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ATTRACTOR FOR THE DISSIPATIVE GENERALIZED KLEIN-GORDON-SCHRÖDINGER EQUATIONS*

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Abstract In this paper the authors consider the Cauchy problem of dissipative generalized Klein-Gordon-Schrödinger equations and prove the existence of the maximal attractor in the weak topology sense.

Key Words Dissipative generalized Klein-Gordon-Schrödinger equations; bounded absorbing set; weak compactness; maximal attractor.

Classification 35Q55, 35Q53, 58F39.

1. Introduction

In this paper we consider the following dissipative generalized Klein-Gordon-Schrödinger equations (GKGS)

$$i\psi_t + \Delta\psi + F_1(|\psi|^2, \phi)\psi + i\alpha\psi = f(x), \qquad t > 0,\ x \in \mathbf{R} \tag{1.1}$$

$$\phi_{tt} + (1 - \Delta)\phi + \beta\phi_t = F_2(|\psi|^2, \phi) + g(x), \tag{1.2}$$

supplemented with initial conditions

$$\psi(0, x) = \psi_0(x), \quad \phi(0, x) = \phi_0(x), \quad \phi_t(0, x) = \phi_1(x), \quad x \in \mathbf{R} \tag{1.3}$$

where ψ and ϕ are unknown complex-valued and real-valued functions respectively, α and β are positive constants, $\Delta = \dfrac{\partial^2}{\partial x^2}$, $(F_1(u, v), F_2(u, v)) = \nabla F(u, v)$, where $F(u, v)$

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is a smooth function from $\mathbf{R}^+ \times \mathbf{R}$ into $\mathbf{R}$ with $F(0,0) = 0$. When $F(u,v) = uv$, i.e. $F_1(|\psi|^2, \phi) = \phi$, $F_2(|\psi|^2, \phi) = |\psi|^2$ in (1.1) (1.2), the system describes the interaction of complex nucleon field with real neutral meson field through Yukawa coupling.

For the classical conservative KGS, i.e. $\alpha = \beta = 0$, $f = g = 0$ and $F(u,v) = uv$, the system has been studied by many authors. [1] proved the existence of global solutions by using the $L^p - L^q$ estimates for the elementary solutions of the Schrödinger equation. [2] and [3] studied the asymptotic behavior of the solutions for the Cauchy problem. In [4] the authors discussed the initial boundary value problem and obtained the global existence of strong solutions in the three-dimensional case, and later the results were improved in [5].

When $F(u,v) = uv$, the system (1.1) (1.2) has been studied by [6] [7] in the case of a bounded domain Ω and by [8] in the case of an entire space $\mathbf{R}^n$. In [6] P. Biler proved the existence of the maximal attractor in the weak topology of the phase space $H^1 \times H^1 \times L^2(\Omega)$. In [7] Li presented a decomposition of the semigroup $S(t)$ and proved that the attractor exists in the norm topology of the phase space $H^2 \times H^2 \times H^1(\Omega)$. In the case of entire space, the imbedding of $H^s(\mathbf{R}^n)$ into $H^{s'}(\mathbf{R}^n)$ $(s > s')$ is not compact, so the decomposition in [7] is not suitable for this situation. In [8] the authors adapted the decomposition in [9] and [10] and showed the asymptotic smoothness of $S(t)$ (cf. [11]) and thus proved the existence of a maximal attractor in $H^2 \times H^2 \times H^1(\mathbf{R}^n)$ which attracts bounded sets of $H^3 \times H^3 \times H^2(\mathbf{R}^n)$ $(n = 3)$.

In the present paper we shall prove, that under certain conditions on the general F, the system (1.1) (1.2) possesses a maximal attractor in the weak topology of $H^1 \times H^1 \times L^2(\mathbf{R})$. We first establish some time-uniform *a priori* estimates on the solutions of (1.1) (1.2), then we show the unique existence of the solution. We shall directly prove the weak continuity of semigroup $S(t)$, which implies the weak compactness of $S(t)$ and thus implies the existence of the maximal weak compact attractor in $H^1 \times H^1 \times L^2(\mathbf{R})$.

We introduce the following standard notations. We denote the spaces of complex valued functions and real valued functions by the same symbols. For $s \geq 0$, $1 \leq p \leq \infty$, $H^{s,p}(\mathbf{R})$ is the usual Sobolev spaces of orders s. $H^s(\mathbf{R}) = H^{s,2}(\mathbf{R})$. $\langle \cdot, \cdot \rangle$ denotes the dual product of $H^{-1}(\mathbf{R})$ and $H^1(\mathbf{R})$. We denote by $\| \cdot \|_p$ the norm of $L^p(\mathbf{R})$ and by $\| \cdot \|_{s,p}$ the norm of $H^{s,p}(\mathbf{R})$. Especially $\| \cdot \| = \| \cdot \|_2$. C is a generic constant and may assume various values from line to line.

2. *A Priori* Estimates and Unique Existence of the Solution

In this section we shall first establish some time-uniform *a priori* estimates on (ψ, ϕ, ϕ_t) in the phase space $V = H^1 \times H^1 \times L^2(\mathbf{R})$ and then make use of them to show