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GLOBAL SMOOTH SOLUTIONS AND THE ASYMPTOTIC BEHAVIOR OF THE MOTION OF A VISCOUS, HEAT-CONDUCTIVE, ONE-DIMENSIONAL REAL GAS

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Abstract The system of balance laws of mass, momentum and energy for a viscous, heat-conductive, one-dimensional real gas is considered. The existence of globally defined smooth solution to an initial boundary value problem is established. Because of the boundary conditions' effect, vacuum will be developped as time tends to infinity.
Key Words Global (local) existence; a priori estimate; the asymptotic behavior.
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1. Introduction

In this paper, we consider the global existence and the asymptotic behavior of smooth solutions to initial boundary value problems in the dynamics of a one-dimensional, viscous, heat-conductive real gas. The referential (Lagrangian) form of the conservation laws of mass, momentum and energy is

$$\begin{cases} u_t - v_x = 0 \\ v_t - \sigma_x = 0 \\ \left(e + \dfrac{v^2}{2}\right)_t - (\sigma v)_x + q_x = 0 \end{cases} \tag{1.1}$$

while the second law of thermodynamics is expressed by Clausius-Duhem inequality

$$\eta_t + \left(\frac{q}{\theta}\right)_x \geq 0 \tag{1.2}$$

Here $u, v, \sigma, e, \eta, \theta$ and q denote the specific volume, the velocity, the stress, the specific internal energy, the specific entropy, the temperature and the heat flux, respectively. Note that u, e and θ may only take positive values.

We shall consider the system (1.1) in the region $\{0 \le x \le 1, t \ge 0\}$ under the initial conditions and the boundary conditions:

$$u(x,0) = u_0(x) > 0, \quad v(x,0) = v_0(x), \quad \theta(x,0) = \theta_0(x) > 0 \tag{1.3}$$

$$q(0,t) = q(1,t) = 0, \quad t \ge 0 \tag{1.4}$$

$$v(0,t) = \sigma(0,t), \quad \sigma(1,t) = 0, \quad t \ge 0 \tag{1.5}$$

The condition (1.4) implies that the ends are thermally insulated. (1.5) means that one end of the gas is put in a vacuum while the other is connected to some sort of dash pot. Here we consider the Newtonial fluid:

$$\sigma(u, \theta, v_x) = -p(u, \theta) + \frac{\mu(u, \theta)}{u} v_x \tag{1.6}$$

satisfying the Fourier law of the heat flux

$$q(u, \theta, \theta_x) = -\frac{k(u, \theta)}{u} \theta_x \tag{1.7}$$

where the internal energy e and the pressure p are interrelated by

$$e_u(u, \theta) = -p(u, \theta) + \theta p_\theta(u, \theta) \tag{1.8}$$

in order to comply with (1.2). This model can be found in [1] and [2].

We assume that e, p, σ and k are twice continuously differentiable on $0 < u < \infty$, and $0 \le \theta < \infty$. As regards growth with respect to the temperature we require that there are exponents $r \in [0,1]$, $s \ge 1 + r$ and positive constants ν, p_1, k_0, and for any $\underline{u} > 0$ there are positive constant $N(\underline{u}), p_2(\underline{u})$, and $k_1(\underline{u})$ such that for $u \ge \underline{u}$ and $\theta \ge 0$, the following conditions hold:

$$0 \le e(u,0), \quad \nu(1 + \theta^r) \le e_\theta(u, \theta) \le N(\underline{u})(1 + \theta^r) \tag{1.9}$$

$$p_1(l + (1-l)\theta + \theta^{1+r}) \le pu \le p_2(\underline{u})(l + (1-l)\theta + \theta^{1+r}), \quad l = 0 \text{ or } 1 \tag{1.10}$$

$$p_u \le 0 \tag{1.11}$$

$$|p_u| \le N(\underline{u})(1 + \theta^{1+r}), \quad |p_\theta| \le N(\underline{u})(1 + \theta^r) \tag{1.12}$$

$$k_0(1 + \theta^s) \le k(u, \theta) \le k_1(\underline{u})(1 + \theta^s) \tag{1.13}$$

$$|k_u(u, \theta)| + |k_{uu}(u, \theta)| \le k_1(\underline{u})(1 + \theta^s) \tag{1.14}$$

The above growth conditions are motivated by the facts in [1] and [2], where it is pointed out that e grows as θ^{1+r} with $r \approx 0.5$ and k increases like θ^s with $s \in [4, 5, 5.5]$. Note that for an ideal gas, $r = 0$, and $l = 0$, where

$$e = c\theta, \quad \sigma = -R\frac{\theta}{u} + \mu\frac{v_x}{u}, \quad q = -k\frac{\theta_x}{u} \tag{1.15}$$